Hartree-Fock-Bogoliubov theory of dipolar Fermi gases

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We apply the Hartree-Fock-Bogoliubov theory to a system of uniform dipolar fermionic polar molecules, which recently has attracted much attention due to rapid experimental progress in achieving such systems. By calculating the anisotropic superfluid-order parameter and the critical temperature T_c, we show that high-T_c superfluid can be achieved with a quite modest value of interaction strength for polar molecules. In addition, we also show that the presence of the Fock-exchange interaction enhances superfluid pairing.

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I. INTRODUCTION

Recent experimental progress in ultracold polar molecules [1] has generated great interest for studying the properties and the applications of such systems. Applications associated with the internal energy levels of polar molecules range from quantum-information processing [2] to spin-model engineering [3]. An equally intriguing direction is focusing on the external degrees of freedom [4]: The system of ultracold polar fermionic molecules with permanent electric dipoles represents an ideal setup for studying dipolar effects in quantum-degenerate fermions [5–7], as the dipolar interaction strength in these molecular systems is several orders of magnitude larger than that in atomic ones.

Notably, two fundamental properties of the dipolar Fermi gases are superfluid pairing [8,9] and Fermi surface deformation [10,11], which are induced by the partially attractive nature of the dipolar interaction and the anisotropic Fock-exchange interaction, respectively. Mathematically, the long-range interaction greatly complicates the calculation. As a result, pioneering works such as in Refs. [8,10,11] concentrated on each of these two features and also made further approximations for simplicity. A quantitatively reliable fully self-consistent calculation that includes both these features is lacking.

The goal of the present work is to fill, at least on the mean-field level (which is believed to be reliable at low temperatures for three-dimensional systems), this gap. In order to achieve this, we apply the Hartree-Fock-Bogoliubov theory that takes account of the interaction effects in a self-consistent manner. We show how this theory can be efficiently implemented by numerically calculating the superfluid-order parameter and the critical temperature T_c for superfluid transition. From our results, we show that a robust superfluid (with T_c as a significant fraction of the Fermi temperature) can easily be reached with ultracold polar molecules. We also investigated the interplay between Fermi surface deformation and superfluid pairing and show that the Fock-exchange interaction leads to anisotropic momentum distribution and, at the same time, enhances superfluid pairing via modifying the density of states near the deformed Fermi surface.

The remainder of this paper is organized as follows. In Sec. II, the general formalism of the Hartree-Fock-Bogoliubov theory, which describes a spinless Fermi system is discussed. This general theory is applied to a uniform dipolar Fermi gas in Sec. III. This yields a set of coupled equations that need to be solved self-consistently. The results are presented and are discussed in Sec. IV. Finally, a conclusion is given in Sec. V.

II. GENERAL FORMALISM

We consider an ensemble of spinless fermions with a general two-body interaction potential $U(\mathbf{r},\mathbf{r}') = U(\mathbf{r}',\mathbf{r})$ confined in an external trapping potential $V(\mathbf{r})$. The second quantized Hamiltonian reads

$$H = \int d\mathbf{r}\,\psi^\dagger(\mathbf{r})\left[-\frac{\hbar^2\nabla^2}{2m} - \mu + V(\mathbf{r})\right]\psi(\mathbf{r}) + \frac{1}{2}\int d\mathbf{r}\int d\mathbf{r}'\,\psi^\dagger(\mathbf{r})\psi^\dagger(\mathbf{r}')U(\mathbf{r},\mathbf{r}')\psi(\mathbf{r}')\psi(\mathbf{r}), \quad (1)$$

where ψ is the fermion field operator, and μ is the chemical potential. By denoting $\{\eta(\mathbf{r})\}$ as a complete set of single-particle eigenstates of $-\hbar^2\nabla^2/(2m) + V(\mathbf{r})$ with eigenenergies ε^0_η, and the associated annihilation operator C_η, Hamiltonian Eq. (1) can be rewritten as

$$H = \sum_\eta \varepsilon_\eta C^\dagger_\eta C_\eta + \frac{1}{2}\sum_{\eta_1,\eta_2,\eta_3,\eta_4} U_{\eta_1,\eta_2,\eta_3,\eta_4} C^\dagger_{\eta_1} C^\dagger_{\eta_2} C_{\eta_3} C_{\eta_4},$$

where $\varepsilon_\eta \equiv \varepsilon^0_\eta - \mu$, and

$$U_{\eta_1,\eta_2,\eta_3,\eta_4} = \int d\mathbf{r}\int d\mathbf{r}'\,\eta^*_1(\mathbf{r})\eta^*_2(\mathbf{r}')U(\mathbf{r},\mathbf{r}')\eta_3(\mathbf{r}')\eta_4(\mathbf{r}).$$

By performing the mean-field decoupling on the quartic operators following the standard procedure [12], we obtain the effective mean-field Hamiltonian,

$$H_{\text{eff}} = \sum_\eta \varepsilon^0_\eta C^\dagger_\eta C_\eta + \sum_{\eta,\eta'}\Big\{[U_h(\eta,\eta') + U_f(\eta,\eta')]C^\dagger_\eta C_{\eta'} - \frac{1}{2}[U_h(\eta,\eta') + U_f(\eta,\eta')]\langle C^\dagger_\eta C_{\eta'}\rangle - \frac{1}{2}\Delta(\eta,\eta')\langle C^\dagger_\eta C^\dagger_{\eta'}\rangle + \frac{1}{2}\Delta(\eta,\eta')C^\dagger_\eta C^\dagger_{\eta'} + \frac{1}{2}\Delta^*(\eta,\eta')C_{\eta'}C_\eta\Big\},$$

where the Hartree term U_h, the Fock term U_f, and the pairing term (or the superfluid-order parameter) Δ are defined as

$$U_h(\eta,\eta') = \sum_{\eta_1,\eta_2} \langle \eta_1,\eta|U|\eta',\eta_2\rangle \langle C^\dagger_{\eta_1} C_{\eta_2}\rangle,$$

$$U_f(\eta,\eta') = -\sum_{\eta_1,\eta_2} \langle \eta_1,\eta|U|\eta_2,\eta'\rangle \langle C^\dagger_{\eta_1} C_{\eta_2}\rangle,$$

$$\Delta(\eta,\eta') = \sum\nolimits_{\eta_1,\eta_2} \langle \eta,\eta'|U|\eta_1,\eta_2\rangle \langle C_{\eta_1} C_{\eta_2}\rangle.$$

$H_{\rm eff}$ has a quadratic form and can, therefore, be diagonalized by using the standard Bogoliubov transformation.

III. APPLICATION TO DIPOLAR FERMI GAS

We now apply the general theory outlined earlier to a system of uniform dipolar Fermi gas with dipole moment $\mathbf{d} = d\hat{z}$ polarized along the z axis. Such a system can be realized by using, for example, ultracold polar molecules. It is convenient to study this problem in momentum space. Instead of η, we use the momentum $\mathbf{k}$ to label the single-particle states with $\varepsilon^0_k = \hbar^2 k^2/(2m)$. The interaction potential in momentum space is given by [13]

$$U(\mathbf{q}) = (4\pi/3)\, d^2(3\,\cos^2\,\theta_{\mathbf{q}} - 1), \tag{2}$$

where $\theta_{\mathbf{q}}$ is the angle between $\mathbf{q}$ and the z axis.

From the symmetry of the system, at least for an interaction strength that is not too strong, we anticipate that pairing only occurs between a particle with momentum $\mathbf{k}$ and another particle with momentum $-\mathbf{k}$ [14]. In other words, the ground state has the usual BCS form:

$$|\text{g.s.}\rangle = \prod_{\mathbf{k}} (u_{\mathbf{k}} + v_{\mathbf{k}} C^\dagger_{\mathbf{k}} C^\dagger_{-\mathbf{k}})|\text{vacuum}\rangle.$$

Consistent with this ground state, the effective Hamiltonian can be written as

$$H_{\rm eff} = \frac{1}{2}[C^\dagger_{\mathbf{k}} \quad C_{-\mathbf{k}}] \begin{bmatrix} \epsilon(\mathbf{k}) & \Delta(\mathbf{k}) \\ \Delta^*(\mathbf{k}) & -\epsilon(\mathbf{k}) \end{bmatrix} \begin{bmatrix} C_{\mathbf{k}} \\ C^\dagger_{-\mathbf{k}} \end{bmatrix} + E_0, \tag{3}$$

where $E_0 = \frac{1}{2}\sum_k [\epsilon(\mathbf{k}) - U_f(\mathbf{k})\langle C^\dagger_{\mathbf{k}} C_{\mathbf{k}}\rangle - \Delta(\mathbf{k})\langle C^\dagger_{\mathbf{k}} C^\dagger_{-\mathbf{k}}\rangle]$,

$$\epsilon(\mathbf{k}) = \varepsilon_k + U_f(\mathbf{k}), \tag{4}$$

$$U_f(\mathbf{k}) = -\sum_{\mathbf{k}'} U(\mathbf{k}'-\mathbf{k})\langle C^\dagger_{\mathbf{k}'} C_{\mathbf{k}'}\rangle, \tag{5}$$

$$\Delta(\mathbf{k}) = \sum_{\mathbf{k}'} U(\mathbf{k}-\mathbf{k}')\langle C_{-\mathbf{k}'} C_{\mathbf{k}'}\rangle. \tag{6}$$

Note that the Hartree term $U_h(\mathbf{k}) = U(0)\sum_{\mathbf{k}'}\langle C^\dagger_{\mathbf{k}'} C_{\mathbf{k}'}\rangle$ vanishes as, for dipolar interaction, $U(0) = 0$. In addition, it is easy to see that $U_f(\mathbf{k}) = U_f(-\mathbf{k})$ and $\Delta(\mathbf{k}) = -\Delta(-\mathbf{k})$.

The effective Hamiltonian Eq. (3) takes the diagonalized form

$$H_{\rm eff} = \frac{1}{2}\sum_{\mathbf{k}} [E(\mathbf{k})\gamma^\dagger_{\mathbf{k}}\gamma_{\mathbf{k}} - E(\mathbf{k})\gamma_{-\mathbf{k}}\gamma^\dagger_{-\mathbf{k}}] + E_0, \tag{7}$$

in terms of the quasiparticle operators,

$$\begin{bmatrix} \gamma_{\mathbf{k}} \\ \gamma_{\mathbf{k}} \end{bmatrix} = \begin{bmatrix} u^*_{\mathbf{k}} & v^*_{\mathbf{k}} \\ -v_{\mathbf{k}} & u_{\mathbf{k}} \end{bmatrix} \begin{bmatrix} C_{\mathbf{k}} \\ C^\dagger_{\mathbf{k}} \end{bmatrix},$$

with

$$u^2_{\mathbf{k}} = \frac{1}{2}\left[1 + \frac{\epsilon(\mathbf{k})}{E(\mathbf{k})}\right], \quad v^2_{\mathbf{k}} = \frac{1}{2}\left[1 - \frac{\epsilon(\mathbf{k})}{E(\mathbf{k})}\right],$$

where $E(\mathbf{k}) = \sqrt{\epsilon(\mathbf{k})^2 + |\Delta(\mathbf{k})|^2}$ represents the quasiparticle dispersion relation.

We remark that the quasiparticle dispersion $E(\mathbf{k})$ may appear to have a similar form as that in the usual BCS theory for a two-component Fermi system with contact interaction. There is, however, a notable difference: In the usual BCS theory, the Hartree-Fock term is ignored as it represents a constant energy shift and can be absorbed into the definition of the chemical potential. Here, in contrast, the Hartree-Fock contribution (for the uniform system considered here, only the Fock term survives) is anisotropic, due to the anisotropy of the dipolar interaction, and must be included explicitly. In fact, even for a quite modest dipolar interaction strength, the Fock term has important effects and can lead to a rather significant deformation of the Fermi surface [10,11].

At thermal equilibrium, we have $\langle \gamma^+_{\mathbf{k}}\gamma_{\mathbf{k}}\rangle = f(E(\mathbf{k}))$, $\langle \gamma_{-\mathbf{k}}\gamma^+_{-\mathbf{k}}\rangle = 1 - f(E(\mathbf{k}))$, where $f(x) = 1/(1+e^{\beta x})$ is the Fermi-Dirac distribution function. Consequently, the self-energy term, Eq. (4) and the pairing term, Eq. (6) take the following forms:

$$\epsilon(\mathbf{k}) = \varepsilon_k - \sum\nolimits_{\mathbf{k}'} U(\mathbf{k}'-\mathbf{k})\left[\frac{1}{2} - \frac{\epsilon(\mathbf{k}')}{2E(\mathbf{k}')}\tanh\frac{\beta E(\mathbf{k}')}{2}\right],$$

$$\Delta(\mathbf{k}) = -\sum\nolimits_{\mathbf{k}'} U(\mathbf{k}-\mathbf{k}')\frac{\Delta(\mathbf{k}')}{2E(\mathbf{k}')}\tanh\frac{\beta E(\mathbf{k}')}{2}. \tag{8}$$

It is known that the gap equation is ultraviolet divergent. The origin of the divergence can be attributed to the fact that the dipolar interaction potential used here [Eq. (2)] is not valid for a large momentum. For a large momentum, or equivalently, for a short distance, the dipolar interaction potential should be significantly modified due to repulsion between electrons. Just as in the treatment of a two-component Fermi gas with contact interaction, we need to regularize the interaction in the gap equation. This problem has been investigated by Baranov and co-workers [8]. In short, the bare dipolar interaction potential $U(\mathbf{k}-\mathbf{k}')$ in the pairing term should be replaced by the vertex function

$$\Gamma(\mathbf{k}-\mathbf{k}') = U(\mathbf{k}-\mathbf{k}') - \sum_{\mathbf{q}} \Gamma(\mathbf{k}-\mathbf{q})\frac{1}{2\varepsilon^0_q} U(\mathbf{q}-\mathbf{k}'),$$

and the gap equation should be renormalized as

$$\Delta(\mathbf{k}) = -\sum_{\mathbf{k}'} \Gamma(\mathbf{k}-\mathbf{k}')\Delta(\mathbf{k}')\left\{\frac{\tanh\left[\frac{\beta E(\mathbf{k}')}{2}\right]}{2E(\mathbf{k}')} - \frac{1}{2\varepsilon^0_{k'}}\right\}. \tag{9}$$

Equations (8) and (9), together with the number equation $N = \sum_{\mathbf{k}} n(\mathbf{k})$, where

$$n(\mathbf{k}) = |u_{\mathbf{k}}|^2 f(E(\mathbf{k})) + |v_{\mathbf{k}}|^2[1 - f(E(\mathbf{k}))]$$

is the momentum distribution function, comprise a complete description of the dipolar Fermi gas and need to be solved self-consistently.

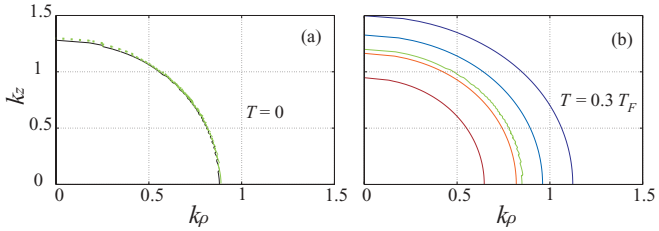

FIG. 1. (Color online) Contour of the momentum distribution function $n(\mathbf{k})$ for $C_{dd} = 1$ at temperatures (a) $T = 0$ and (b) $T = 0.3\ T_F$. In (a), we draw the Fermi surface. The solid line is the Fermi surface obtained from the self-consistent calculation of this work, the dotted line is the one obtained from the variational approach developed in Refs. [10,11]. In (b), the lines from outside to inside correspond to $n(\mathbf{k}) = 0.1$, 0.3, 0.5, 0.7, and 0.9, respectively. Here, and in other plots, $\mathbf{k}$ is in units of the Fermi wave number of the noninteracting system $k_F = (6\pi^2 n)^{1/3}$.

IV. RESULTS

Now we present some results. First, we neglect superfluid pairing and focus on the deformation of the Fermi surface in a normal state. Then, we discuss the combined effects of Fermi surface deformation and superfluidity.

A. Deformation of Fermi surface in a normal state

First, let us consider a normal dipolar gas by taking $\Delta(\mathbf{k}) = 0$. Note that $\Delta = 0$ is always a solution to the gap equation, Eq. (9). Figure 1 illustrates the momentum distribution function $n(\mathbf{k})$ as a function of k_z and $k_\rho = \sqrt{k_x^2 + k_y^2}$, for two different temperatures. The dipolar interaction strength is fixed at $C_{dd} = 1$ where $C_{dd} = m\, d^2 n^{1/3}/\hbar^2$ is the dimensionless dipolar strength [11]. $C_{dd} = 1$ corresponds to the RbK molecule created at the Joint Institute for Laboratory Astrophysics experiment at a modest density of about $4 \times 10^{-12}\ \mathrm{cm}^{-3}$. Furthermore, at this value of C_{dd}, one can estimate, by using the results of Ref. [11], that the ratio of the interaction energy is about 20% of the Fermi energy; hence, a mean-field calculation should be quantitatively accurate. At $T = 0$, $n(\mathbf{k}) = v_{\mathbf{k}}^2 = \Theta(-\epsilon(\mathbf{k}))$, where $\Theta(\cdot)$ is the step function. In Fig. 1(a), we draw the contour of the Fermi surface. In Refs. [10,11], we developed a variational approach and assumed that the Fermi surface of the dipolar gas had an ellipsoidal shape,

$$n(\mathbf{k}) = \Theta\left(1 - \alpha^2 k_z^2 - k_\rho^2/\alpha\right),$$

where α is the variational parameter that characterizes the deformation of the Fermi surface. At $C_{dd} = 1$, we obtain $\alpha = 0.7769$. In Fig. 1(a), the dotted line represents the contour of the Fermi surface from this variational calculation. As one can see, the variational result matches very well with the full self-consistent calculation. At a larger C_{dd}, a small difference can be see between the two results. In general, the variational results exhibit slightly stronger deformation. The same conclusion has been reached by Ronen and Bohn [15]. Figure 1(b) shows the momentum distribution at $T = 0.3T_F$, $n(\mathbf{k}) = 0.1$, 0.3, 0.5, 0.7, and 0.9, respectively, where $T_F = E_F/k_B$ is the Fermi temperature of the noninteracting system. At finite temperature, Fermi surface gets smeared out. However, the anisotropy of the momentum distribution is still quite clear and should be observable in the expansion dynamics [11].

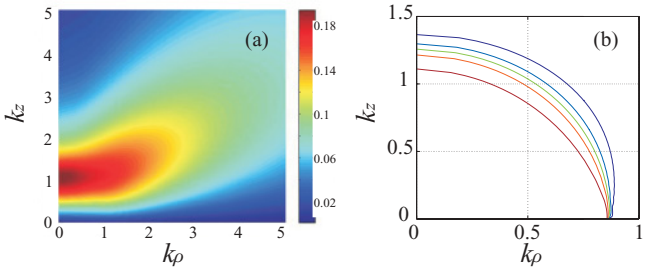

FIG. 2. (Color online) (a) Gap parameter $\Delta(\mathbf{k})$ [in units of $E_F = \hbar^2 k_F^2/(2m)$] at $T = 0$ for $C_{dd} = 1$. (b) The corresponding contour plot of the momentum distribution function $n(\mathbf{k})$. The lines from outside to inside correspond to $n(\mathbf{k}) = 0.1$, 0.3, 0.5, 0.7, and 0.9, respectively.

B. Dipolar superfluidity

Let us now turn to the discussion of the superfluid state. For simplicity, we take the first-order Born approximation by replacing the vertex function $\Gamma(\mathbf{k} - \mathbf{k}')$ in the gap equation, Eq. (9) by the bare dipolar interaction $U(\mathbf{k} - \mathbf{k}')$. This is equivalent to introducing an energy cutoff in the momentum summation and is expected to be a good approximation as long as the dipolar interaction strength is not too strong [8].[1] In Fig. 2(a), we plot the zero-temperature gap parameter $\Delta(\mathbf{k})$ for $C_{dd} = 1$. $\Delta(\mathbf{k})$ is an odd function of $\mathbf{k}$ and vanishes for $k_z = 0$. As a consequence, the Fermi surface smears out except at $k_z = 0$, as can be seen from the momentum distribution shown in Fig. 2(b). The peak value of Δ nearly reaches $0.2E_F$ for this rather modest dipolar interaction strength, and occurs near $k_z = k_F$ and $k_\rho = 0$. To investigate the angular distribution of Δ, we note that

$$\Delta(\mathbf{k}) = \Delta(k, \cos\theta_{\mathbf{k}}) = \sum_{\text{odd } l} \Delta_l(k) Y_{l0}(\cos\theta_{\mathbf{k}}),$$

where $k = |\mathbf{k}|$ and, due to the cylindrical symmetry of the system, only odd l and $m = 0$ components are present. In Fig. 3, we plot $\Delta_l(k)$ for $C_{dd} = 1$. For small values of k ($k \lesssim k_F$), Δ is dominated by the $l = 1$ (p-wave) component. For larger k, contribution from higher partial waves may become important.

Next, we illustrate the finite-temperature effects in Fig. 4. The dashed line in Fig. 4(a) represents the chemical potential of the superfluid state as a function of temperature. It increases with temperature. In comparison, the chemical potential of the normal state [the solid line in Fig. 4(a)] is a monotonically decreasing function of temperature [16]. Figure 4(b) shows how T_c varies with C_{dd}. The solid line is a fit according to

$$T_c/T_F = a\ \exp(-b/C_{dd}), \tag{10}$$

[1]Here, for the typical dimensionless dipolar strength, we used $C_{dd} = 1$. By using the variational method of Refs. [10,11], we can estimate that the interaction energy is about 20% Fermi energy.

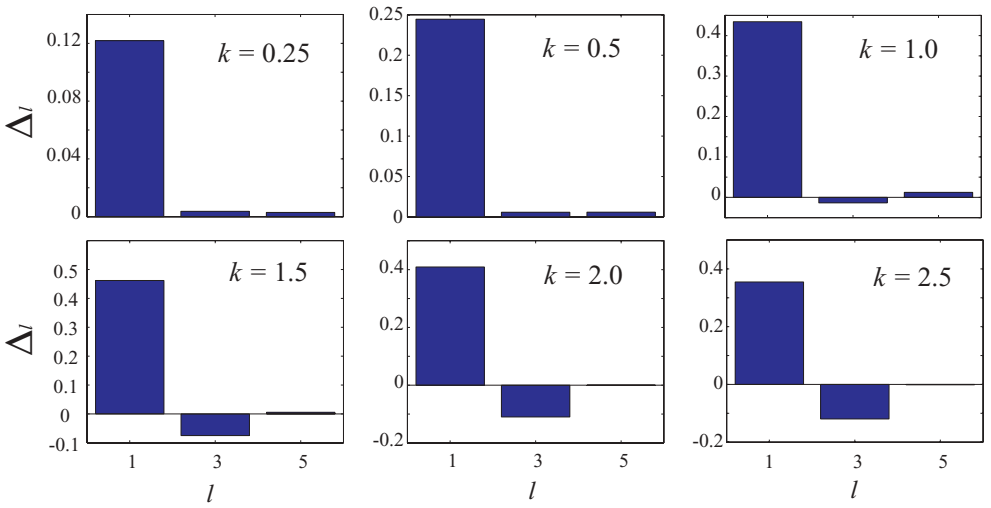

FIG. 3. (Color online) $\Delta_l(k)$ for $C_{dd} = 1$ at zero temperature.

with the fitting parameters $a = 0.8363$ (0.5444,1.128) and $b = 2.194$ (1.778,2.61), where the values quoted in the parentheses represent the 95% confidence bound.

Some remarks are in order. First, from our calculation, we find that $T_c \approx 0.1T_F$ at $C_{dd} = 1$. If we were dealing with a two-component Fermi gas with contact interaction, such a critical temperature would correspond to a system inside the unitary regime. As we have mentioned, $C_{dd} = 1$ is a quite modest value for polar molecules. Therefore, typical polar molecules can easily reach the strongly interacting regime. Second, it is imperative to compare Eq. (10) to the critical temperature found by Baranov *et al.* [8], which, in our notation, takes the form

$$T_c/T_F \approx 1.44 \exp\left[-\frac{\pi^3}{4(6\pi^2)^{1/3}C_{dd}}\right] = 1.44 \exp(-1.9887/C_{dd}).$$

One can notice that the coefficients in the exponent agree quite well. Less agreement is found in the prefactor. This is, however, understandable, as there are several differences in our treatment. For example, Baranov *et al.* have included beyond-mean-field fluctuations and the contribution from the second-order Born approximation [17], and fixed the chemical potential to be at the Fermi energy as in standard weakly interacting theories, while neglecting the Fock term in their calculation. For the range of dipolar strength plotted in Fig. 4(b), the T_c obtained in our calculation is lower than that reported in Ref. [8] by about a factor of 2. This discrepancy may be reduced if higher-order terms in the vertex function are included [17]. However, by going beyond the first-order Born approximation in a self-consistent calculation such as we did here is extremely complicated.[2] On one hand, given the very different approaches employed and the different approximations invoked in the two works, this difference is not unacceptable; on the other hand, this also indicates that a numerically accurate calculation of T_c for a dipolar Fermi gas is rather difficult.

Finally, to reveal the interplay between pairing and Fermi surface deformation, we artificially turn off the Fock term in our calculation. We find that the presence of the Fock-exchange interaction increases both the critical temperature and the magnitude of the order parameter by $20 \sim 25\%$. This enhancement can be understood in the following way. The presence of the Fock term causes an ellipsoidal deformation of the Fermi surface in such a way that it stretches the momentum

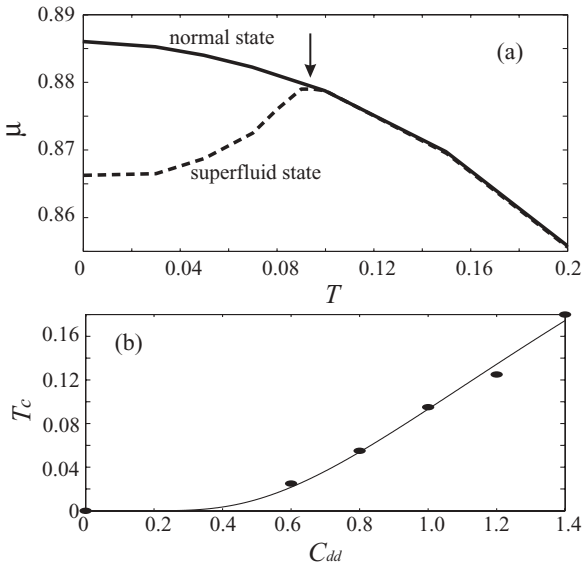

FIG. 4. (a) Chemical potential μ as a function of temperature for a superfluid state (dashed line) and a normal state (solid line). Energy and temperature are in units of E_F and T_F, respectively. Here, $C_{dd} = 1$. The arrow indicates the location of T_c. A superfluid state exists for $0 < T < T_c$. (b) T_c as a function of C_{dd}. Here, the dots are numerical data, and the smooth curve is a fit according to Eq. (10).

[2]Several recent works concerning superfluid states of two-component dipolar Fermi gas [18], and one-component dipolar Fermi gas in quasi-two-dimensional geometry [9,19] also adopted first-order Born approximation.

distribution along the z axis. As a result, the density of states near the Fermi surface is increased along z and is reduced along the transverse directions. On the other hand, the dipolar-induced pairing is dominated by the p-wave symmetry (i.e., strongest in the z direction). Therefore, the Fock interaction-induced Fermi surface deformation tends to enhance superfluid pairing.

V. CONCLUSION

We have presented a self-consistent calculation under the framework of the Hartree-Fock-Bogoliubov formalism to study a system of spinless fermions with long-range interaction. This theory is analogous to the generalized BCS theory developed by Leggett for the study of BCS-Bose-Einstein condensation crossover [20]. However, here, we also include the Hartree-Fock interaction term, which, along with the chemical potential, must be calculated self-consistently. We applied this theory to describe an ensemble of uniform polar Fermi molecules, and we calculated the superfluid-order parameter and the critical temperature T_c. Both our work and that of Ref. [8] show that a typical Fermi gas of polar molecules can easily reach the strongly interacting regime with T_c as a significant fraction of the Fermi temperature T_F. However, a numerical discrepancy of T_c between the two approaches also indicates the difficulty in obtaining a quantitatively accurate estimate of T_c. Our work further shows that the Fock interaction, in addition to deforming the Fermi surface, has the effect of enhancing superfluid pairing. In the future, it will be of great interest to investigate the collective excitations of the superfluid dipolar Fermi gases, the effects of quantum fluctuations, and the possibility of novel quantum phases that may arise at a large dipolar interaction strength and/or in the presence of an optical lattice potential [21].

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