Quantum phase transition in an array of coupled dissipative cavities

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We study the features of superfluidand Mott-insulator states in a two-dimensional array of cavities with a two-level atom embedded in each cavity which is strongly coupled to the cavity field and immersed in a bosonic bath. Employing a different quasiboson approach, we show analytically how the dissipation and decoherence influence the quantum phase transitions from two aspects and find a localized tendency. For the superfluid state, a dynamical instability will lead to a sweeping to a localized state of photons. For the Mott-insulator state, a dissipation-induced fluctuation will suppress the restoring of long-range phase coherence driven by interaction.

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Introduction. One of the remarkable applications of coupled cavity arrays is to realize quantum simulators. Due to the controllability of atomic and optical systems, it could be useful to attack some unclear physics and to explore new phenomena in quantum many-body systems [1–4]. In particular, recent experimental progress in the control of optical systems [5–8] and in the fabrication of large-scale arrays of high-Q cavities [9,10] make these potential applications close to becoming a reality. Moreover, theoretical proposals, including paradigms such as the Bose-Hubbard model and effective spin model [11–13], have been put forward and have predicted a large quantity of novel applications [14–17]. However, the quantum optical system is typically driven by an external laser source and is always coupled to its environment [18–20]. These result in effects such as dissipation, decoherence, and entanglement [21], and bring the system out of equilibrium [22–29] and profoundly affect the dynamics of interest [30–33]. Important questions thus arise and need to be clarified, e.g., under realistic experimental conditions, how would the dissipation and decoherence behave in these open systems?

In this paper, we propose answers to the above question by investigating the superfluid(SF)–Mott-insulator phase transition in the array of dissipative coupled cavities. We show that the transition shares some of the features of the nondissipative counterparts. There are still two quantum many-body states that can be recognized as the delocalized and localized states of photons. However, very differently, the dissipation and the decoherence give rise to a localizing effect and drive the system into mixed states. For the superfluid state, a nonequilibrium dynamical instability can lead to a sweeping to a localized state at a finite time. For the Mott-insulator state, where photons are already localized at each lattice site, the localization holds, but a dissipation-induced fluctuation of photon number acting on each lattice site will suppress the restoration of long-range phase coherence.

Model. Consider a system consisting of atoms and cavities coupled weakly to a bosonic environment at zero temperature. As the size of the individual cavities is generally much smaller than their spacing, we assume that the photons emitted from each cavity are uncorrelated. The total Hamiltonian therefore reads

$$H = H_s + H_{\text{bath}} + H_{\text{coup}}, \tag{1}$$

where H_s is the Hamiltonian for the system, $H_{\text{bath}} = \sum_j \sum_{\alpha,k} \omega_{k_\alpha} r^\dagger_{j,k_\alpha} r_{j,k_\alpha}$ is the Hamiltonian for the environment, and $H_{\text{coup}} = \sum_j \sum_{\alpha,k} (\eta^*_{k_\alpha} r^\dagger_{j,k_\alpha} \alpha_j + \text{H.c.})$ is the coupled term. In addition, $\alpha = a, c$ labels the operators and physical quantities associated with atoms and cavities, respectively; ω_{k_α} denotes the frequency of environmental modes; $r^\dagger_{j,k_\alpha}$ and $r_{j,k_\alpha}$are the creation and annihilation operators of quanta in the k_αth model on the jth lattice site; and η_{k_α} is the coupling strength. Here we set $\hbar = 1$.

The system we modeled, as depicted in Fig. 1, is a two-dimensional array of resonant optical cavities, each embedded with a two-level (artificial) atom coupled strongly to the cavity field. The possible realizations include photonic band-gap cavities and superconducting strip line resonators [4]. With ω_a and ω_c being the frequency of atom transition and cavity mode, respectively, in the rotating wave approximation (RWA), such an individual atom-cavity system on site j is well described by the Jaynes-Cummings Hamiltonian, $H^{\text{JC}}_j = \omega_a a^\dagger_j a_j + \omega_c c^\dagger_j c_j + \beta(a^\dagger_j c_j + \text{H.c.})$. Here $a^\dagger_j$ and a_j ($c^\dagger_j$, c_j) are atomic (photonic) raising and lowering operators, respectively, and β is the coupled strength. In the grand canonical approach, H_s is therefore given by combining H^{JC}_j with the photonic hopping term and chemical potential term,

$$H_s = \sum_j H^{\text{JC}}_j - \sum_{\langle j,j'\rangle} \kappa_{jj'} c^\dagger_j c_{j'} - \sum_j \mu n_j. \tag{2}$$

Here $\kappa_{jj'}$ is the photonic hopping rate between cavities. Since the evanescent coupling between cavities decreases with the distance exponentially, we restrict the summation $\sum_{\langle j,j'\rangle}$ running over the nearest neighbors. $n_j = a^\dagger_j a_j + c^\dagger_j c_j$ is the total number of atomic and photonic excitations on site j. μ is the chemical potential, where the assumption $\mu = \mu_j$ for all sites has been made.

Due to the strong coupling, as shown in Fig. 1(b), the resonant frequencies of the individual atom-cavity system are split into $E_{|\pm,n\rangle} = n\omega_c \pm \sqrt{n\beta^2 + \frac{\Delta^2}{4}} - \frac{\Delta}{2}$, where $|\pm,n\rangle$

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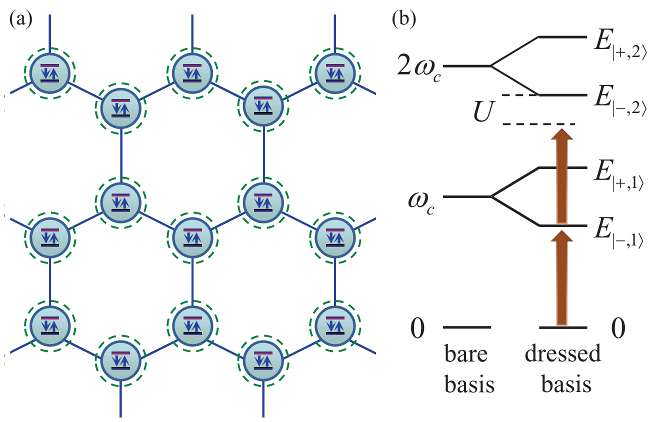

FIG. 1. (Color online) A type of possible topologies for two-dimensional cavity arrays for z nearest neighbors. (a) Individual cavities are coupled resonantly to each other due to the overlap of the evanescent fields. Each cavity contains a two-level system coupled strongly to the cavity field and immersed in a bosonic bath (marked by the dash line). (b) Energy eigenvalues of the individual cavity-atom system on each site. $\omega_c = \omega_a$ is assumed for simplicity. The anharmonicity of the Jaynes-Cummings energy levels can effectively provide an on-site repulsion U to block the absorption for the next photon.

labels the positive (negative) branch of dressed states, and $\Delta = \omega_c - \omega_a$ is the detuning. The anharmonicity of the Jaynes-Cummings energy levels can effectively provide an on-site repulsion. For instance, the resonant excitation by a photon with frequency $E_{|\pm,1\rangle}$ will prevent the absorption of a second photon at $E_{|\pm,1\rangle}$, which is the striking effect known as photon blockade [8]. It is therefore feasible to realize a quantum simulator in terms of the system described by Eq. (2). This so-called Jaynes-Cummings-Hubbard (JCH) model was recently suggested by Greentree *et al.* [2].

Methods. However, the situation changes dramatically once the degrees of freedom of the environment are taken into consideration, as described by Hamiltonian (1). The nonequilibrium dynamics for the open quantum many-body system will arise, which is a formidable task to solve. Here we propose a treatment to eliminate those external degrees of freedom. To approach this, we rewrite Hamiltonian (1) as

$$H = H_{\text{local}} - \sum_{\langle j,j'\rangle} \kappa_{jj'} c_j^\dagger c_{j'} - \sum_j \mu n_j, \tag{3}$$

where $H_{\text{local}} = \sum_j H_j^{\text{JC}} + H_{\text{bath}} + H_{\text{coup}}$.

First, considering the case where the jth cavity containing an initial photon interacts with a bath, the dynamics is governed by

$$H_j = \omega_c c_j^\dagger c_j + \sum_k \omega_{k_c} r_{j,k_c}^\dagger r_{j,k_c} + \sum_k \left(\eta_{k_c}^* r_{j,k_c}^\dagger c_j + \text{H.c.}\right). \tag{4}$$

We denote its eigenvalue as ω and expand the eigenvector $|\phi_j\rangle$ as $|\phi_j\rangle = e_c c_j^\dagger |\emptyset\rangle + \sum_k e_k r_{k_c}^\dagger |\emptyset\rangle$. Here e_c and e_k are the probability amplitudes for the excitation occupied by the cavity field and the environment, respectively. $|\emptyset\rangle$ denotes the vacuum state. Deducing the equations of these two amplitudes, one can express e_k in terms of e_c and, under the Born-Markov approximation, integrate out the degrees of freedom of the environment, and obtain $(\omega_c + \delta\omega_c - i\gamma_c) e_c = \omega e_c$. $\delta\omega_c$ is known as an analog to the Lamb shift in atomic physics and is significantly small when the coupling to the environment is weak. γ_c is the decay rate and indicates a finite lifetime of the cavity mode [34].

This motivates us to introduce a quasiboson, described by C_j, with a complex eigenfrequency $\Omega_c = \omega_c - i\gamma_c$, where $\delta\omega_c$ has been absorbed into ω_c, to redescribe the cavity field coupled with a bath in terms of $H_j^{\text{eff}}|\phi_j\rangle = \Omega_c|\phi_j\rangle$. $H_j^{\text{eff}} = \Omega_c C_j^\dagger C_j$ is the effective Hamiltonian, and now $|\phi_j\rangle = e_c C_j^\dagger|\emptyset\rangle$. Because of loss, the system would be nonconservative and the corresponding operators would be non-Hermitian. The commutation relation of C_j reads $[C_j, C_{j'}^\dagger] = (1 + i\frac{\gamma_c}{\omega_c})\delta_{jj'}$, in which $\frac{\gamma_c}{\omega_c}$ is of the order of $\frac{1}{Q}$, with Q being the quality factor of the individual cavity. The bosonic commutation relation is therefore approximately satisfied for the high-Q cavity, which can be met in most cavity quantum electrodynamics (QED) experiments.

The complex eigenfrequency underlines the facts that, on one hand, dissipation is an inherent property for a realistic cavity. When a photon with a certain frequency has been injected into a dissipative cavity, the composite system of the cavity field plus the environment cannot be characterized merely by the frequency of the injected photon, because we must take the impacts of the environment into account. On the other hand, in general we are not concerned with the time evolution of the bath. In this way, the array of dissipative cavities can be regarded as a configuration consisting of quasibosons. Quite similar operations can be performed on an atom to introduce another kind of quasiboson described by A_j with frequency $\Omega_a = \omega_a - i\gamma_a$, where γ_a is the atomic decay rate.

We can therefore rephrase Hamiltonian (1) with the renormalized terms,

$$H = \sum_j H_j^{\text{eff}} - \sum_{<j,j'>} \kappa_{jj'} C_j^\dagger C_{j'} - \sum_j \mu n_j, \tag{5}$$

where now $H_j^{\text{eff}} = \Omega_a A_j^\dagger A_j + \Omega_c C_j^\dagger C_j + \beta(A_j^\dagger C_j + \text{H.c.})$ and $N_j = A_j^\dagger A_j + C_j^\dagger C_j$. One nice feature of Hamiltonian (5) here is that the losses are described by leaky rates γ_a and γ_c rather than by operators. Without having to mention the external degrees of freedom, this effective treatment would be of great conceptual and, moreover, computational advantage, rather than the general treatment as Hamiltonian (1). A more microscopic consideration points out that in the cavity QED region, since the atom is dressed by the cavity field, the atom and field act as a whole subjected to a total decay rate Γ [35]. Specifically, $\Gamma = n(\gamma_a + \gamma_c)$ for $\Delta = 0$.

To gain insight into the role of dissipation in the SF–Mott-insulator phase transition, we use a mean-field approximation, which could give reliable results if the system is at least two dimensional [36]. We introduce a superfluid parameter, $\psi = \text{Re}\langle C_j\rangle = \text{Re}\langle C_j^\dagger\rangle$. In the present case, the expected value of C_j ($C_j^\dagger$) is in general complex with the formation $\langle C_j\rangle = \psi - i\psi_\gamma$ ($\langle C_j^\dagger\rangle = \psi + i\psi_\gamma$). ψ_γ is a solvable small quantity as a function of γ_a and γ_c, and vanishes in the limit of no loss.

By using the decoupling approximation $C_j^\dagger C_{j'} = \langle C_j^\dagger\rangle C_{j'} + \langle C_{j'}\rangle C_j^\dagger - \langle C_j^\dagger\rangle\langle C_j^\dagger\rangle$, the resulting mean-field Hamiltonian can be written as a sum over a single site,

$$H_{\rm MF} = \sum_j \left[H_j^{\rm eff} - z\kappa\psi(C_j^\dagger + C_j) + z\kappa|\psi|^2 - \mu N_j + O(\psi_\gamma^2)\right], \tag{6}$$

where we have set the intercavity hopping rate $\kappa_{jj'} = \kappa$ for all nearest neighbors, with z labeling the number.

ψ can be examined analytically in terms of the second-order perturbation theory, with respect to the damped dressed basis. For energetic favor, we assume each site is prepared in the negative branch of the dressed state. But because the dressed basis is defined on $n \geqslant 1$, a ground state $|0\rangle$ with the energy $E_{|0\rangle} = 0$ needs to be supplemented. Thus

$$\psi = e^{-\Gamma t}\sqrt{-\frac{\chi}{z\kappa\Theta}}. \tag{7}$$

χ and Θ are functions of all of the parameters for the whole system. Since the evanescent parameter κ is a typical small quantity in systems of coupled cavities, the perturbation theory gives good qualitative and quantitative descriptions compared to the numerical results given by explicitly diagonalizing [2,37].

Arguably the most interesting situation is the case in which the effective photon-photon interactions are maximized, namely, when cavities are on resonant with atoms and with one initial excitation per site [38]. In addition, $\Gamma = \gamma_a + \gamma_c = \gamma$. With $F_1 = \omega_c - \beta - \mu$ and $F_2 = -\omega_c + (\sqrt{2}-1)\beta + \mu$, in Eq. (7), $\Theta = \frac{1}{2F_1^2+2\gamma^2} + \frac{3+2\sqrt{2}}{4F_2^2+4\gamma^2} > 0$, and

$$\chi = \frac{F_1}{2F_1^2+2\gamma^2} + \frac{(3+2\sqrt{2})F_2}{4F_2^2+4\gamma^2} + \frac{1}{z\kappa e^{-2\gamma t}}. \tag{8}$$

In the absence of loss, one can recognize that $\chi = 0$ is the well-known, self-consistent equation, and therefore distinguish the SF phase and Mott-insulator phase. Nevertheless, the coupling to the environment induces a nonequilibrium dynamics, thus no strictly defined phase exists. However, if the external time dependence is much slower than the internal frequencies of the system, then there remain two fundamentally different quantum states that can be identified by whether ψ vanishes or has a finite value, i.e., photons localized in each lattice site (Mott-insulator-like state) and delocalized across the cavities (SF-like state).

Analyses. To analyze the physics of the transition between these two states in detail, we proceed with our discussion from two aspects. First, we start with the superfluid phase and track the time evolution of the long-range phase coherence. The prefactor $e^{-\Gamma t}$ in Eq. (7) indicates the expected decay of ψ. However, more importantly, a dynamical instability due to the coupling to the external environment is revealed by χ. As illustrated in Fig. 2, for $t \ll \beta^{-1}$, ψ has a slight reduction scaled by $\frac{\gamma^2}{\beta^2}$. For $t > \beta^{-1}$, $z\kappa e^{-2\gamma t}$ is the leading term and pronounces the decrease of the effective tunneling energy. Consequently, a photon hopping rate κ given initially in the superfluid region will cross the critical point at a time $t_c \simeq \frac{1}{2\gamma}\ln\frac{\kappa}{\kappa_c}$, with κ_c being the critical tunneling energy for the nondissipative case with a given z and β. Before t_c, a nonlocal region is still recognized as nonlocal. The dissipation has not changed the fundamental nature of the system, albeit with the reduction of long-range phase coherence and an additional fluctuation due to photon leakage. Nevertheless, beyond t_c, the superfluidity breaks down, i.e., a sweeping to the localized state does occur [39]. An analogous localizing effect was described in an optical lattice system very recently, where the spontaneous emission of atoms owing to the lattice heat led to decoherence of the many-body state [40].

In what follows, in contrast, we start in the Mott-insulator state and discuss the impacts of dissipation on the critical behavior and the fluctuation behavior. Consider that the initial

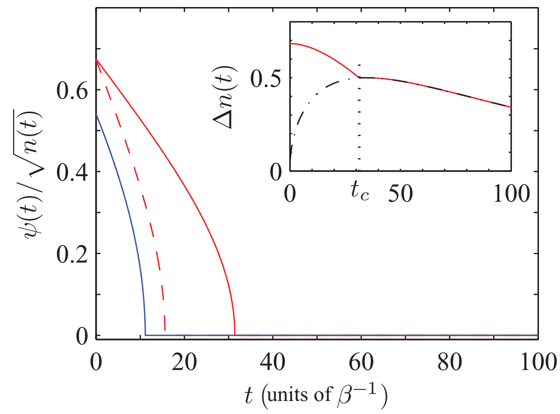

FIG. 2. (Color online) The temporal decrease of the superfluidity and the photon number fluctuation on each site for a certain initial state (inset). For a initial SF state, the long-range order decays continuously and the fluctuation on each site is a total effect of the photon hopping and the photon leakage before t_c [from left to right $(\frac{z\kappa}{\beta},\frac{\gamma}{\beta}) = (0.2,0.01),(0.3,0.02)$ and $(0.3,0.01)$ respectively, in the main panel]. The superfluidity breaks down and the related photon number fluctuation behaves as the fluctuation of a Mott-insulator-like state beyond t_c [the solid line for $(\frac{z\kappa}{\beta},\frac{\gamma}{\beta}) = (0.3,0.01)$ and the dot-dashed line for $(\frac{z\kappa}{\beta},\frac{\gamma}{\beta}) = (0,0.01)$ in inset].

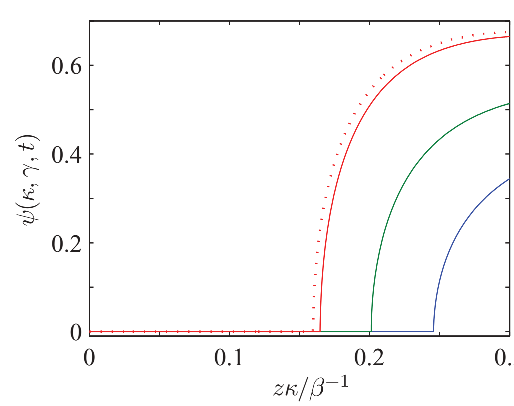

FIG. 3. (Color online) The restoring of long-range phase coherence from the Mott-insulator state. Influences of dissipation depend on the leaky rate γ (the dotted and solid lines for $\frac{\gamma}{\beta} = 0$ and 0.05, respectively) and will accumulate along with time [the red, green, and blue solid lines (left to right) for $t = 0$, $0.1\gamma^{-1}$, and $0.2\gamma^{-1}$, respectively, with t in units of β^{-1}].

state is deep in the Mott-insulator phase, $\frac{z\kappa}{\beta} = 0$, and we continuously increase the intercavity coupled rate. For the related ideal case, one can reach the superfluid phase at $\frac{z\kappa}{\beta} = (\frac{z\kappa}{\beta})'_c \simeq 0.16$. However, the presence of a bath converts coherences originally in the system into entanglement of the system and the environment [6], thus the effective tunneling energy will be lower than expected. Moreover, this impact will continue to accumulate over time. As shown in Fig. 3, to expect the appearance of photonic hopping, we must keep increasing κ. On the other hand, although long-range order is still absent, which is different from the pure Mott-insulator state, there will be a fluctuation owing to photon leakage acting on each lattice site (dot-dashed line in Fig. 2). Consequently, it will not be able to restore the long-range phase coherence perfectly by the driven $\frac{z\kappa}{\beta}$ into the superfluid region.

Conclusions. In summary, we have shown analytically the features of the superfluid–Mott-insulator phase transition in the array of dissipative cavities. Our analysis sufficiently takes into account the intrinsically dissipative nature of an open quantum many-body system, and identifies how dissipation and decoherence would come into play. For the further experimental signature, we predict that there will be a localizing effect.

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