Diffraction of Bose-Einstein condensates in quantized light fields

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We investigate the atomic diffraction of Bose-Einstein condensates in quantized light fields. Situations in which the light fields are in number states or coherent states are studied theoretically. Analytical derivation and numerical calculation are carried out to simulate the dynamics of the atomic motion. In the condition that atoms are scattered by light in the number states with imbalanced photon-number distribution, the atomic transitions between different momentum modes would sensitively depend on the transition order and the photon-number distribution. The number-state nature of the light fields modifies the period of atomic momentum oscillations and makes forward and backward atomic transitions unequal. For light fields in coherent states, whether or not the intensities of the light fields are balanced, the atomic diffraction is symmetric and independent of the transition order.

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I. INTRODUCTION

The coherent interaction between matter and electromagnetic fields within various kinds of physical systems is providing a useful platform for developing concepts in quantum optics and atomic and molecular physics. The experimental realization of Bose-Einstein condensation in dilute atomic gases [1] greatly facilitates the investigations of the interactions between atoms and light on a macroscopic scale [2–6]. In the past few years, the atomic scattering processes in optical fields have been intensively studied, such as matter-wave supperradiance [7–9], quantum phase transition [10,11], quantum tunneling and measurement [12,13], and cavity optomechanics [14,15], raising a variety of striking discoveries.

In physical systems involving light-atom interactions, various kinds of experimental configurations have been developed or proposed to achieve the desired atom-field coupling [16,17]. Particularly, by using high-quality resonators [18–20], the regime of strong coupling between the light and the atoms can be reached, where atoms coherently exchange photons with light fields. However, in the previous light-atom-interaction models, the derivations usually used the classical treatment for the electromagnetic fields, in which the light fields are treated as amplitude-modulated plane waves with slowly varying amplitudes; even in a quantized treatment for light fields, mean-field approximation is often introduced for the light fields. Atomic diffraction in such situations has been well studied; however, in most cases, the subtle effects of the atomic motion induced by the quantum nature of light have always been ignored.

One of our previous work was devoted to investigating the atomic interaction effects of a Bose-Einstein condensate (BEC) in classical light fields [17]. In this paper, we study the diffraction of a BEC in multimode optical fields, with a full quantum treatment of the light fields. In the cases when the quantized light fields are in different kinds of states, exotic behaviors are induced in the atomic motion by the quantum nature of light. This paper is organized as follows. In Sec. II, we introduce the physical system and develop the theoretical model of the atomic diffraction in a full quantum treatment of the light fields. In Sec. III, we analyze how the quantum status of the pump fields can modify the atomic motion in the cases when the optical fields are in number states. In Sec. IV, we turn to the case where the light fields are in coherent states. In Sec. V, we study the interaction effects raised from the atomic interactions within the condensate. In Sec. VI, we arrive at a summary of our results.

II. THE THEORETICAL MODEL FOR ATOMIC DIFFRACTION IN QUANTIZED LIGHT FIELDS

The physical system being studied is illustrated in Fig. 1, where an elongated BEC is trapped along the horizontal axis of a triangular ring cavity. The cavity consists of three mirrors that are placed in a way such that the light reflected inside the cavity forms a closed loop. There are two mutually counter-propagating modes, i.e., the clockwise and counterclockwise modes, coupled equally to the atoms. The light fields in these two modes are also referred to as forward (or right-going, with wave vector k_R) and backward (or left-going, with wave vector k_L), according to the geometrical configuration. These two cavity modes are degenerated in frequency, $\omega_R = \omega_L = \omega_c$, so the wave vectors of the two cavity modes satisfy $k_R = -k_L$. The atoms within the condensate are recognized as two-level atoms with one internal ground state $|g_{\rm in}\rangle$ and an excited state $|e_{\rm in}\rangle$ representing the internal degrees of freedom for each atom. In the large detuning limit, the upper internal state $|e_{\rm in}\rangle$ of the atoms can be adiabatically eliminated [9,21,22]. For illustration, the condensate is prepared with ^{87}Rb atoms, and the wave length of the light fields is $\lambda = 780$ nm with a detuning $\Delta = \omega_c - \omega_a = -1.5$ GHz of the D_2 transition [8,16,17].

The Hamiltonian of the light-atom coupling system can be written as

$$\mathscr{H} = \int d^3r\,\hat{\Psi}^\dagger(\mathbf{r})\left[-\frac{\hbar^2}{2M}\nabla^2 + \hat{\mathbf{E}}(\mathbf{r})\cdot\hat{\mathbf{D}}(\mathbf{r})\right]\hat{\Psi}(\mathbf{r}) + \frac{1}{2}\int d^3r d^3r'\,\hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}^\dagger(\mathbf{r}')V_{\rm inter}(\mathbf{r}-\mathbf{r}')\hat{\Psi}(\mathbf{r}')\hat{\Psi}(\mathbf{r}), \tag{1}$$

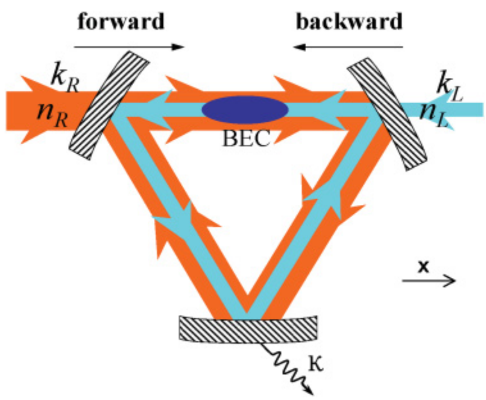

FIG. 1. (Color online) A schematic diagram of a two-mode triangular ring cavity containing an elongated Bose-Einstein condensate trapped along one of its cavity axes. The damping rate κ of the cavity modes is small compared to the atomic motion inside the cavity. There are two degenerate light modes propagating in the clockwise and counterclockwise directions inside the cavity.

where $\hat{\Psi}^\dagger(\mathbf{r})$ and $\hat{\Psi}(\mathbf{r})$ are the creation and annihilation field operators for the atomic field, respectively. $\hat{\mathbf{E}}(\mathbf{r})$ is the electric field operator of the electromagnetic field, and $\hat{\mathbf{D}}(\mathbf{r})$ is the atomic dipole moment operator. $V_{\text{inter}}(\mathbf{r}-\mathbf{r}')$ represents the interatomic potential. The counterpropagating light fields form a standing wave in the triangular ring cavity. The atoms in the standing light feel a periodical potential. The condensate is transversely tightly bounded; therefore its transverse freedom is frozen, and it is reasonable to recognize the condensate as a quasi-one-dimensional ensemble in its longitudinal direction [23]. This suggests that the matter field operators can be expanded with a set of Bloch functions, which are amplitude-modulated plane waves,

$$\hat{\Psi}(\mathbf{x}) = \sum_n \hat{b}_n e^{i\mathbf{k}_n\cdot\mathbf{x}} u(\mathbf{x}),$$
$$\hat{\Psi}^\dagger(\mathbf{x}) = \sum_n \hat{b}_n^\dagger e^{-i\mathbf{k}_n\cdot\mathbf{x}} u^*(\mathbf{x}), \quad (2)$$

where $u(\mathbf{x})$ is a periodic function along the direction of the horizontal axis of the cavity, $u(x+\frac{\lambda}{2}) = u(x)$, with the period one-half of the wave length of the light wave. $\hat{b}_n^\dagger$ and $\hat{b}_n$ are the creation and annihilation operators of the bosonic atoms in the nth Bloch mode, respectively. $\mathbf{k}_n = 2n\mathbf{k}_R$ is the wave vector of the modulated plane wave. For the absorption and subsequently stimulated emission of photons from the two modes of optical fields, the atoms would acquire a net momentum of even-number multiples of photon momentum, $2n\hbar k_R$. Assuming the atoms are initially in the stationary state, after the scattering process, they will be excited to the higher adjacent momentum modes, which can be described as $(e^{2i\mathbf{k}_R\cdot\mathbf{x}} + e^{-2i\mathbf{k}_R\cdot\mathbf{x}})u_0(\mathbf{x})$, with $u_0(\mathbf{x})$ being the envelop of the stationary wave packet. If the atoms get further scattered, some of them may go to the even higher momentum mode. In the situations that the atomic interactions in the BEC cannot be neglected, the system will exhibit strong nonlinearity. Using a mean-field approach, Gross and Pitaevskii have independently derived the mean-field equation for the BEC systems [24]. With tunable parameters, the eigenfunctions and dynamics of such systems can be extremely complicated [25,26]. However, if the particle density of the BEC is very low, the atomic interactions of the condensate can be neglected. Then we can approximate the amplitude-modulated plane waves in Eq. (2) with the wave functions of the free particles, and the wave functions of the atomic field now read

$$|\Psi(x)\rangle = \sum_n \Psi_n \left|\frac{1}{\sqrt{V}}\exp\left(\frac{i}{\hbar}p_n x\right)\right\rangle = \sum_n \Psi_n |p_n\rangle,$$
$$\langle\Psi(x)| = \sum_n \Psi_n^* \left\langle\frac{1}{\sqrt{V}}\exp\left(\frac{-i}{\hbar}p_n x\right)\right| = \sum_n \Psi_n^* \langle p_n|. \quad (3)$$

Here V is the volume of the condensate, and $\langle x|p_n\rangle = \frac{1}{\sqrt{V}}\exp(\frac{i}{\hbar}p_n x)$ is the wave function of the free particles with momentum $p_n = 2n\hbar k_R$ in coordinate space. The particle number of the atoms is normalized to 1. At the same time, we have removed the hat from each operator of the matter field, using a mean-field treatment for the atomic field.

We approximate that the discrete modes of the atomic field make a complete set describing the dynamics of the condensate. Plugging Eq. (3) into Eq. (1), we arrive at the following expression for the Hamiltonian of the light-atom coupling system:

$$\hat{H} = \sum_{m=-\infty}^{\infty} \hbar\omega_m |p_m\rangle\langle p_m| + \hbar G \times \sum_{n=-\infty}^{\infty} \left(a_{k_L}^\dagger a_{k_R}|p_n\rangle\langle p_{n-1}| + a_{k_R}^\dagger a_{k_L}|p_n\rangle\langle p_{n+1}|\right), \quad (4)$$

where we have adopted the full quantized version for the electric component of the light fields, $\mathbf{E}(\mathbf{r}) = \sum_{\mathbf{k}} \hat{\epsilon}_{\mathbf{k}} \mathscr{E}_{\mathbf{k}}(a_{\mathbf{k}} + a_{\mathbf{k}}^\dagger), \mathbf{k} = k_{L,R}$, where $\hat{\epsilon}_{\mathbf{k}}$ is the unit polarization vector and $\mathscr{E}_{\mathbf{k}} = \sqrt{\hbar\omega_c/2\epsilon V_c}$ has the dimension of an electric field, with V_c being the quantization volume of the macrocavity [27]. $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^\dagger$ are the creation and annihilation operators of photons in the corresponding light modes, respectively. $\hbar\omega_m = 2(\hbar m k_L)^2/M$ is the energy of the mth unperturbed discretized mode, and M is the mass of a single atom within the condensate. $G = \Omega^2/\triangle$ is the effective two-photon atom-field coupling constant, with $\Omega = -\frac{\mathbf{d}\cdot\hat{\epsilon}_{\mathbf{k}}\mathscr{E}_{\mathbf{k}}}{\hbar}$ being the single-photon atom-field coupling constant and $\mathbf{d}$ being the electric dipole moment of the atom. The form of G comes from the adiabatic elimination of the internal excited state of the two-level atom, and this is legitimated by large detuning of the light fields. This Hamiltonian is rather reminiscent of an ordinary lattice model in solid-state physics in spite of an atomic interacting term. The second hopping term disturbs the motion of the free atoms and induces an exchange of particles between different diffraction orders. Such atomic diffraction is categorized into Raman-Nath diffraction with short pulse fields and Bragg diffraction with a longer time of light-atom interaction.

To investigate the atomic motion of the system, it is usually convenient to work in the interaction picture by separating the Hamiltonian (4) into the free part and the atom-field-interacting part; then one has

$$\hat{\mathscr{V}}(t) = \hbar G \sum_{n=-\infty}^{\infty} \{a_{k_L}^\dagger a_{k_R} |p_n\rangle\langle p_{n-1}| e^{-i\delta_-(t)} + a_{k_R}^\dagger a_{k_L} |p_n\rangle\langle p_{n+1}| e^{-i\delta_+(t)}\}, \quad (5)$$

where $\delta_\pm = \pm 16\pi\nu_r(n \pm \frac{1}{2})$ and $\nu_r = \hbar k_L^2/(4\pi M)$ is the atomic one-photon recoil frequency ($\nu_r \sim 3.77$ kHz according to the parameters chosen in this system, and G is chosen to be $-0.7\nu_r$ in this paper).

The optical freedom of this system can be integrated out to extract the effective information of atomic dynamics. Unlike the classical treatment of the optical fields in which the optical fields are recognized as plane waves with slowly varying amplitudes, the atomic motion of the system can be explicitly modified by the status of the quantized light fields, not just by the strength of the pump beams [16,17]. To see this, we shall assume, for example, that the light fields are in number states. The Hamiltonian (5) conserves the total photon number of the system. Thus the wave function of the matter-wave condensate can be written as a linear combination of different momentum states with the corresponding photon distributions among the two light fields,

$$|\Psi(t)\rangle = \sum_n \Psi_n |\psi(n, N_R - n, N_L + n, t)\rangle,$$

where $n = 0, \pm 1, \pm 2$, refers to the diffraction order. N_R and N_L are the initial photon-number distributions among the two light modes. $|\psi(n, N_R - n, N_L + n, t)\rangle$ is the product of the wave function of the atomic center-of-mass motion and that of the optical fields, $|p_n\rangle \otimes |\psi_f\rangle$. In this stage, $|\psi_f\rangle$ is chosen to be the number state, denoted as $|n_1, n_2\rangle$, indicating that there are n_1 photons in the right-going light mode and n_2 photons in the left-going mode. Since the condensate is initially prepared in the stationary state, each atom needs to absorb n photons from the right-going light mode in order to hop to the nth momentum mode. Therefore, the photon-number distribution of the two light modes is directly related to the atomic diffraction order. The equation of atomic motion now reads

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle &= \hat{\mathscr{V}}(t)|\Psi(t)\rangle \\ &= \hbar G \sum_{n=-\infty}^{\infty} \{a_{k_L}^\dagger a_{k_R} |p_n\rangle\langle p_{n-1}| e^{-i\delta_-(t)} \\ &\quad + a_{k_R}^\dagger a_{k_L} |p_n\rangle\langle p_{n+1}| e^{-i\delta_+(t)}\}|\Psi(t)\rangle. \end{aligned}$$

Keeping the photon degree of freedom, the above equation can be further simplified to

$$\begin{aligned} &\dot{\Psi}_n(t)|n_R - n, n_L + n\rangle \\ &= -iGa_{k_L}^\dagger a_{k_R} \Psi_{n-1} e^{-i\delta_- t} |n_R - n + 1, n_L + n - 1\rangle \\ &\quad -iGa_{k_R}^\dagger a_{k_L} \Psi_{n+1} e^{-i\delta_+ t} |n_R - n - 1, n_L + n + 1\rangle. \end{aligned}$$

Until now, we have kept the quantum character of the optical fields. Integrating the photon degrees of freedom by explicitly executing the operations of the photon creation and annihilation operators on the corresponding number states of the light fields, we arrive at the following equation of the atomic motion:

$$\dot{\Psi}_n(t) = W_R^n \Psi_{n-1} e^{-i\delta_- t} + W_L^n \Psi_{n+1} e^{-i\delta_+ t}, \quad (6)$$

where $W_{L,R}^n$ are the forward and backward transition weights defined as $W_R^n = -iG\sqrt{(N_R - n + 1)(N_L + n)}$ and $W_L^n = -iG\sqrt{(N_R - n)(N_L + n + 1)}$. This equation has included the quantum effect of optical fields upon the atomic motion. The complicated expressions for the transition weights result from the property of the number state of the optical fields that when it is operated by the corresponding creation and annihilation operators, we have $a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$ and $a|n\rangle = \sqrt{n}|n-1\rangle$. It can be directly observed from Eq. (11) that the transition weights depend not only on the diffraction order but also on the photon distribution among the two light modes. Thus, unlike the classical case of plane-wave approximation for the light fields, there is no definite equality between W_R^n and W_L^n; $|W_R^n| > |W_L^n|$ and $|W_R^n| < |W_L^n|$ are both possible, depending on whether $N_R < N_L$ or $N_L > N_R$. Table I lists the transition weights of the first few orders of atomic scattering processes for $N_R = 80$ and $N_L = 10$. It shows that for the first few orders of scattering processes, $|W_R^n| < |W_L^n|$ are always satisfied.

TABLE I. The transition weights depending on the diffraction order in units of the atom-field coupling constant G for the initial photon-number distribution $N_R = 80$, $N_L = 10$.

Scattering Process	$\lvert W_R^n\rvert$	$\lvert W_L^n\rvert$	$\sqrt{\lvert W_R^n W_L^n\rvert}$
$\lvert p_0\rangle \to \lvert p_1\rangle, \lvert p_{-1}\rangle$	28.46	29.66	29.06
$\lvert p_1\rangle \to \lvert p_0\rangle, \lvert p_2\rangle$	29.66	30.79	30.22
$\lvert p_{-1}\rangle \to \lvert p_0\rangle, \lvert p_{-2}\rangle$	27.17	28.46	27.81

III. LIGHT FIELDS IN NUMBER STATES

A. Raman-Nath regime

To solve Eq. (11) in the full time domain, one has to resort to numerical calculations. Fortunately, in the Raman-Nath regime, where the atoms interact with the light fields for a rather short time, the low-order diffraction dominates the whole atomic scattering process, and Eq. (11) can be solved analytically. In this limit $t \to 0$, the probability equation of the atomic motion takes the following form:

$$\dot{\Psi}_n(t) = W_R^n \Psi_{n-1} + W_L^n \Psi_{n+1}. \quad (7)$$

For macroscopic occupations of photons in the two light modes, the diffraction order n is small compared to the initial photon numbers N_R and N_L; i.e., if $N_{R,L} \gg n$, we have $W_R^n \sim W_L^n \sim \sqrt{W_R^0 W_L^0}$, which can be observed in Table I. Thus Eq. (7) has the following solution:

$$\Psi_n(t) = i^n e^{i\theta} \left(\frac{W_R^n}{W_L^n}\right)^{\frac{n}{2}} J_n(\xi t), \quad (8)$$

where $\xi = 2\sqrt{(W_R^n W_L^n)}$, $J_n(\xi t)$ is the nth-order Bessel function, and θ is an arbitrary phase angle. The population of atoms occupying the nth momentum state $|p_n\rangle$ is $\rho_n(t) =$

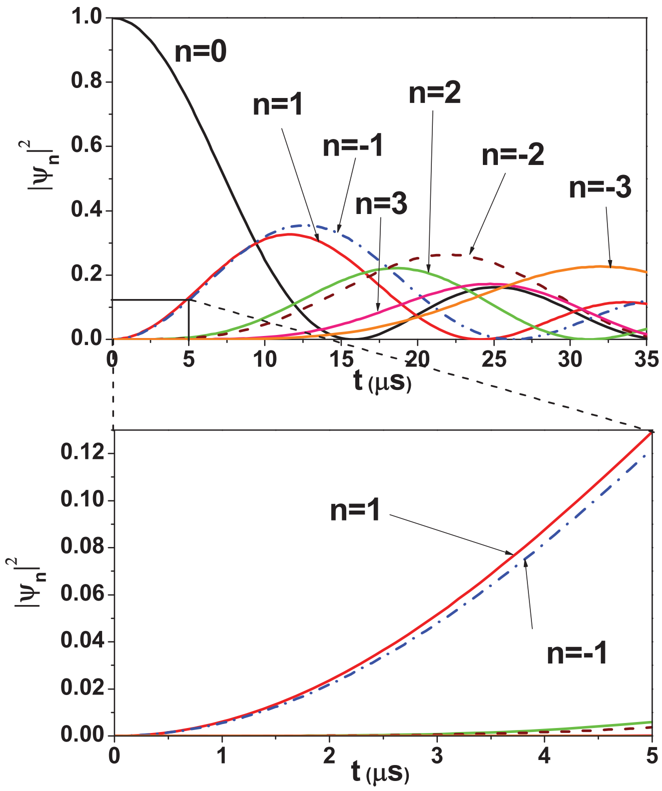

FIG. 2. (Color online) Atomic probability distribution of the first few discrete momentum modes in the limit of short light-atom interacting time for an initial photon-number distribution $N_R = 80, N_L = 10$. Black solid curve, $n = 0$ mode; red solid curve, $n = 1$; blue dash-dotted curve, $n = -1$; green solid curve, $n = 2$; burgundy dashed curve, $n = -2$; pink solid curve, $n = 3$; orange solid curve, $n = -3$. The light-atom coupling constant is chosen as $G = -0.7\nu_r$, with ν_r being the atomic one-photon recoil frequency.

$|\Psi_n(t)|^2 = (\frac{W_R^n}{W_L^n})^n[J_n(\xi t)]^2$. For the inequality of W_R^n and W_L^n, as shown in Table I, the atomic diffraction will display exotic behaviors. Figure 2 shows the probability amplitudes of the atoms in different momentum modes evolving with the light-atom interaction time. It can be seen that, in addition to the atomic populations in different momentum modes oscillating with time, the curves of the momentum oscillations of the $\pm n$th modes do not overlap. Unlike the situations illustrated in Refs. [16,17], the probability amplitudes of these $\pm n$th momentum modes do not reach the minimum or maximum values exactly at the same time. For example, in Fig. 2, there is a shift between the first minima of the probability amplitudes for the ± 1st-order atomic momentum modes. Even though studying the atomic motion for long-period light-atom interaction from the data derived in this regime may not seem adequate enough, it implies that the atomic diffraction would be asymmetric about the zeroth order, and such asymmetries are induced by the number-state nature of the optical fields. A more detailed discussion will be given and the accuracy of the analytical solution in this short-time regime will be demonstrated in the latter part of this paper.

B. Bragg regime

In the Bragg diffraction regime, where the light and atoms interact with each other for a longer time, the exponential factors in Eq. (11) greatly modify the atomic motion, compared to the situations of the short-time light-atom interactions (i.e., Raman-Nath regime). However, in this regime, the equation of atomic motion cannot be explicitly solved analytically. To obtain the information of the atomic motion in the full-time domain, numerical calculation is carried out with a cutoff at the $n = \pm 10$th momentum modes in order to maintain the accuracy. In fact, the validity of such a cutoff has been implied in the case of short-time light-atom interactions; for very high diffraction orders, the atomic occupations in the relevant modes are always vanishingly small.

Because of the dependence of the transition weights upon the diffraction order and the photon-number distribution, numerical simulation displays interesting behaviors of the atomic motion in the quantized light fields. It seems that the atoms can sense the imbalanced intensities of optical fields and thus take action accordingly. Figure 3 shows the probability amplitudes of atoms in the first few orders of momentum modes for different photon number distributions. It is found that, the quasiperiod of momentum oscillations changes with respect to the diffraction order. Figure 3(a) shows the probability amplitudes of the scattered atoms at the photon-number distribution $N_R = 80, N_L = 10$. Even for atomic diffraction of the same order (see the $n = \pm 1$ curves), the atomic populations oscillate at different frequencies. At $t \approx 28\ \mu s$, the curves for the $n = \pm 1$ modes go to their lowest points, but if one reviews the details in this interval, it is found that there is a shift between the minima of two curves. That is to say, the populations of atoms in these two modes are in fact oscillating with different periods. Additionally, it is found that the atoms would favor the forward and backward diffractions alternatively; the $n = 1$ order is sometimes stronger and sometimes weaker than the $n = -1$ order, implying that asymmetry appears in the matter-wave diffraction. This peculiar phenomenon results from the quantum nature of the optical fields. The asymmetric atomic diffraction can be manipulated by adjusting the imbalance of the photon-number distributions in the two light modes. Figure 3(b) shows the atomic distribution of the diffracted atoms for $N_R = 200, N_L = 20$. Similar diffraction behavior is observed as that shown in Fig. 3(a). However, the atomic distributions in this case are oscillating at higher frequencies. Thus the imbalanced photon-number distribution of the light fields modifies the dynamics of the atomic diffraction, and the order dependence of the transition weights induced by the number-state nature of the light fields makes the atomic momentum oscillation very irregular.

The above result is quite different from that obtained from the traditional analysis, where mean-field approximations are applied to both the atomic field and optical fields. The photon creation and annihilation operators are directly replaced with their mean-field values. Even though this is efficient for extracting qualitative information about the atomic dynamics, it ignores the details of influences from the quantum nature of the light fields upon the atomic motion.

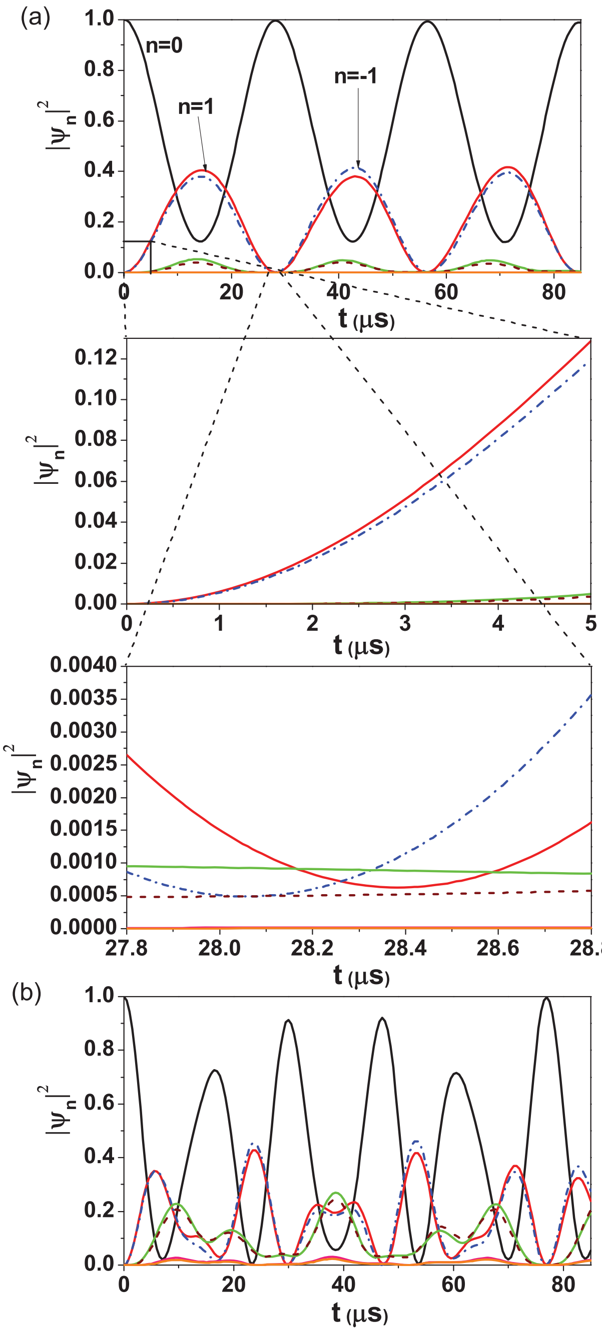

FIG. 3. (Color online) Atomic probability distribution of the first few discrete momentum modes in the regime of long light-atom interacting time for initial photon-number distributions of (a) $N_R = 80, N_L = 10$ and (b) $N_R = 200, N_L = 20$. (Line patterns are the same as in Fig. 2, top.)

To check the accuracy of the solution derived in the short-time limit of the light-atom interactions in the last section, a numerical simulation for Eq. (7) is carried out for the photon-number distribution $N_R = 80, N_L = 10$, and the result is shown in Fig. 4(a). It is found that the data obtained here are consistent with those obtained in the Bragg regime. There are displacements between the minima (or maxima) of the conjugate $\pm n$ curves, and the probability amplitude of the $n = 1$ momentum mode can be either larger or smaller than that of the $n = -1$ mode. This is similar to the result displayed in Fig. 3(a). However, we have to emphasize that using Eq. (7) to describe atomic motion is only valid in the Raman-Nath regime with short-time light-atom interactions; thus data in Fig. 4(a) for large t are not reliable. Figure 4(b) shows the error of the data in this regime compared to that from Fig. 3(a). The accuracy function D_a is defined as $D_a^n(t) = \frac{|\Psi_n^R|^2 - |\Psi_n^B|^2}{|\Psi_n^R|^2 + |\Psi_n^B|^2}$, where Ψ_n^R is the probability amplitude of the atomic motion in the Raman-Nath regime obtained in Sec. III. A and Ψ_n^B is that obtained in the Bragg regime shown in Fig. 3(a). It is found that, for short times, $t < 5\ \mu s$, the error of the analytical solution (8) is less than 1%. So it is a good approximation of the atomic motion in the Raman-Nath regime.

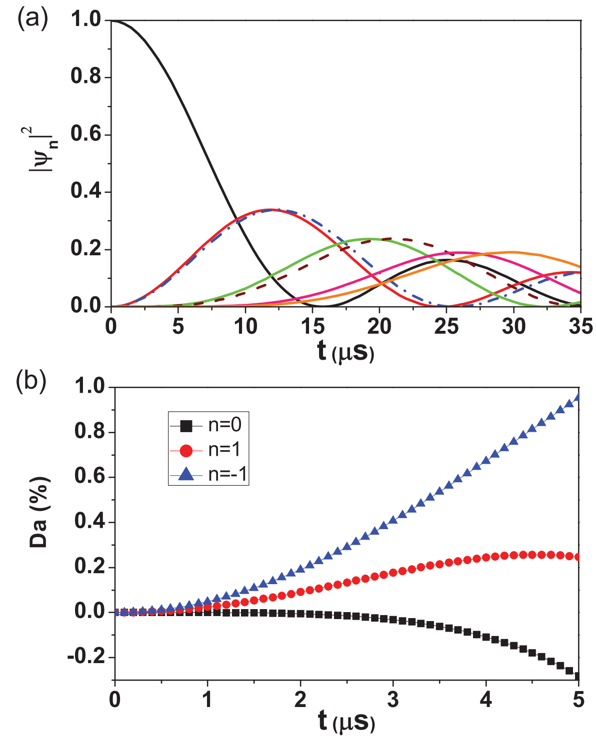

FIG. 4. (Color online) (a) Atomic probability distribution of the first few discrete momentum modes from numerical calculation in the Raman-Nath regime for the photon-number distribution $N_R = 80, N_L = 10$. (Line patterns are the same as in Fig. 2, top.) (b) The error of the analytical solution obtained in the short-time limit of the light-atom interaction compared to that obtained in the Bragg regime.

IV. LIGHT FIELDS IN COHERENT STATES

In addition to the properties of atomic diffraction being sensitively dependent on the intensities of the pump fields, the status of the light fields can also impose implicit corrections to the atomic diffraction. To make a comparison to the case when the lights are in number states, we turn to the case when the two optical fields are in coherent sates $|\alpha_R\rangle$ and $|\alpha_L\rangle$, respectively. α_R and α_L are the eigenvalues of the annihilation operators of the two light modes on the relevant coherent states. The equation of the atomic motion reduces to

$$\dot{\Psi}_n(t) = W_{co}\Psi_{n-1}e^{-i\delta_- t} + W_{co}^*\Psi_{n+1}e^{-i\delta_+ t}, \tag{9}$$

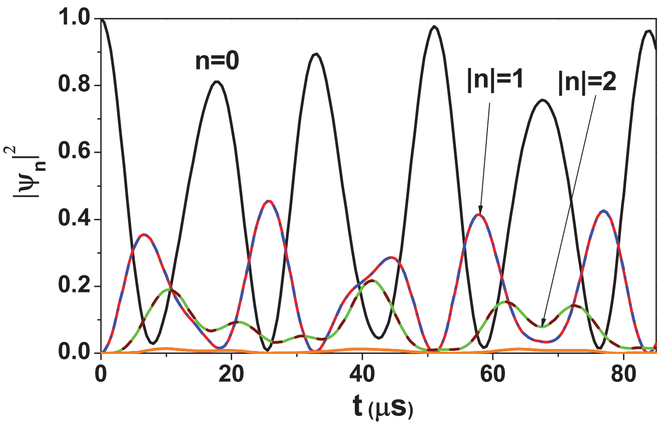

FIG. 5. (Color online) Atomic probability distribution of the first few discrete momentum modes under the condition that the light fields are in coherent sates $|\alpha_R\rangle$ and $|\alpha_L\rangle$. The parameters are chosen such that $\alpha_R\alpha_L^* = 40 + 40i$. (Line patterns are the same as in Fig. 2, top.)

where W_{co} and W_{co}^* are conjugate with each other and $W_{co} = -iG\alpha_R\alpha_L^*$. Generally, α_R and α_L are complex numbers. If α_R and α_L are chosen to be real, then $W_{co} = W_{co}^* = \alpha_R\alpha_L$, and the equation is of the same form as that obtained in Refs. [16,17]. By numerical simulation, the information of the atomic motion is obtained in the full-time domain. Figure 5 shows the probability amplitudes of the first few momentum modes at $\alpha_R\alpha_L^* = 40 + 40i$. It is found that the curves of the $\pm n$th orders coincide with each other; thus the forward and backward atomic transitions are equally favored, and no asymmetry appears in the atomic diffraction. This can be seen from the fact that the absolute values of transition weights for the forward and backward transitions are the same, and their phase difference can be absorbed in the definition of the phase factor $\delta_\pm$ as being constant; thus the equation of the atomic motion still takes the symmetric form like that in Refs. [16,17].

In this case, the transition weights are independent of the diffraction order. The data obtained here resemble those with classical treatments of the optical fields, where the electromagnetic fields are recognized as plane waves. Comparing the results with those in the case when the optical fields are in number states, even though the equation of the atomic motion is also derived in a full quantum treatment of the light fields, only symmetric atomic diffraction can be obtained here; this is determined by the specific status of quantized light fields.

V. INTERACTION EFFECTS

The above discussions apply in the dilute condensate with very weak atomic interactions, in which the nonlinear effects induced by the interactions can be neglected. However, in the situations when the particle density of the BEC is high, the atomic diffraction will be greatly influenced by the atomic interactions. Bragg diffraction of interacting BECs has been studied in Ref. [17], with a classical treatment for the light fields. By taking the atomic interactions into consideration, the Hamiltonian of the light-atom-coupled system with quantized light fields now reads

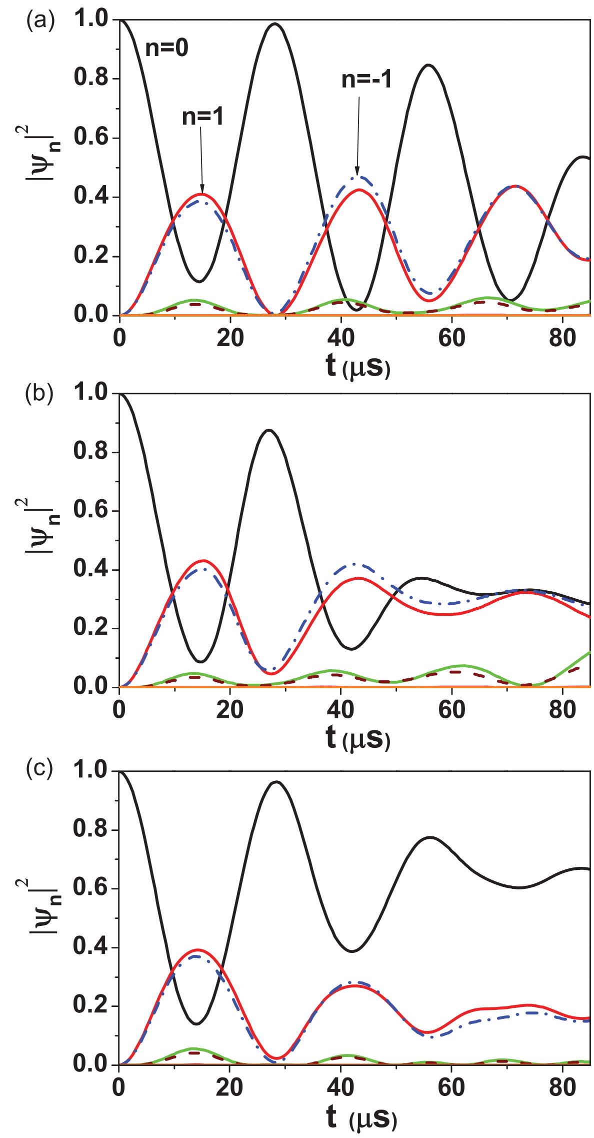

FIG. 6. (Color online) Probability amplitudes of the first few discrete momentum modes with the presence of atomic interactions for the photon-number distribution $N_R = 80, N_L = 10$. (a) Repulsive interaction, $U_0/(\hbar\nu_r) = 1.0$. (b) Repulsive interaction, $U_0/(\hbar\nu_r) = 4.0$. (c) Attractive interaction, $U_0/(\hbar\nu_r) = -2.0$. (Line patterns are the same as in Fig. 2, top.)

$$\hat{H} = \sum_k \hbar\omega_k b_k^\dagger b_k + \frac{1}{2}\sum_{k_1,k_2,q} U(q) b_{k_1+q}^\dagger b_{k_2-q}^\dagger b_{k_1} b_{k_2} + \hbar G \sum_k \{a_{k_L}^\dagger b_{k+1}^\dagger b_k a_{k_R} + a_{k_R}^\dagger b_k^\dagger b_{k+1} a_{k_L}\}, \quad (10)$$

where $b_k^\dagger$ and b_k are the atomic creation and annihilation operators for the kth momentum state, respectively. $U(q)$ is the Fourier transform of the interatomic potential. The atomic interaction conserves the total momentum of the particles participating in the collision. The indexes k,q,k_1, and k_2 run over all the discrete momentum states.

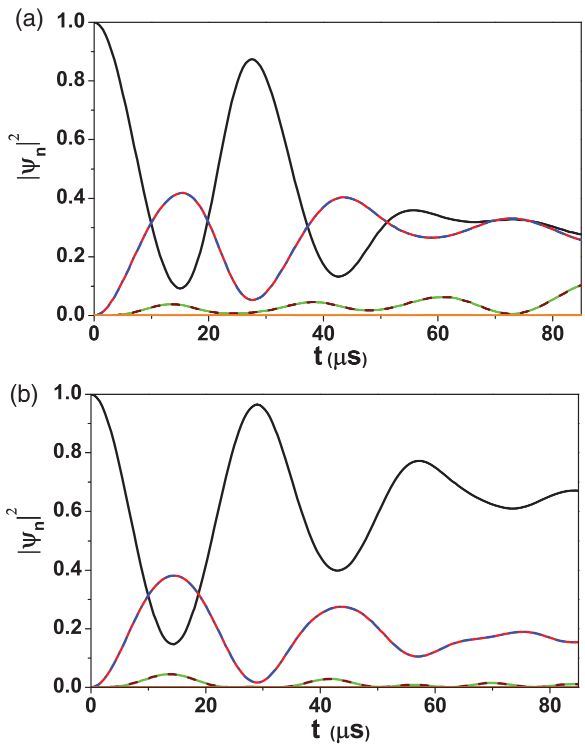

FIG. 7. (Color online) Probability amplitudes of the first few discrete momentum modes with the presence of atomic interactions for light fields in coherent states, $\alpha_R\alpha_L^* = 20 + 20i$. (a) Repulsive interaction, $U_0/(\hbar\nu_r) = 4.0$. (b) Attractive interaction, $U_0/(\hbar\nu_r) = -2.0$. (Line patterns are the same as in Fig. 2, top.)

With the similar method adopted in Ref. [17], the above Hamiltonian can be approached by applying mean-field approximation to the atomic interactions. Finally, we arrive at the following effective equation for the atomic motion, provided that the light fields are in number states:

$$\dot{\Psi}_n(t) = W_R^n \Psi_{n-1} e^{-i\tilde{\delta}_- t} + W_L^n \Psi_{n+1} e^{-i\tilde{\delta}_+ t}, \quad (11)$$

where the operators $\{b_k\}$ in (10) have been replaced with c numbers $\{\Psi_k\}$,

$$\tilde{\delta}_\pm = \delta_\pm + \sum_{q\neq 0}\{U(q) + U(0)\}\{\langle n_{k+q\pm 1}\rangle - \langle n_{k+q}\rangle\}.$$

The shorthand $\langle\cdots\rangle$ means taking the average over the state configuration of the system instantly at each time point.

Here we choose the typical contact interaction, $U(q) = U_0 = \frac{4\pi\hbar^2\rho a}{M}$, with a being the s-wave scattering length and ρ being the particle density of the condensate. Figure 6 shows the probability amplitudes of the first few momentum modes with the presence of the atomic interactions. For the $n = \pm 1$ curves, it is found that the original regular momentum oscillations are dampened by the atomic interactions. This is because the interparticle interactions always tend to smear out the fine collective orders in the many-body systems [17]. According to the results shown in Figs. 6(a) and 6(b), as the atomic interactions becoming stronger, the momentum oscillations of the BEC system die out even more quickly. At the same time, an alternatively favored oscillating behavior can still be observed in the atomic motion according to the parameters chosen in Figs. 6(a), 6(b), and 6(c). This is due to the number-state nature of the pump fields.

For light fields in coherent states, the atomic motion would take a similar symmetric form to that discussed in Sec. IV. The equation of motion for the system can be obtained with

$$\dot{\Psi}_n(t) = W_{co}\Psi_{n-1} e^{-i\tilde{\delta}_- t} + W_{co}^* \Psi_{n+1} e^{-i\tilde{\delta}_+ t}. \quad (12)$$

Figure 7 shows the probability amplitudes of the first few momentum modes in light fields in coherent states, with the presence of the atomic interactions. The curves for the $n = \pm m$ momentum modes coincide with each other. This resembles the classical cases where the light fields are treated as plane waves with slowly varying amplitudes. The reason can also be attributed to those discussed in Sec. IV. The existence of the atomic interactions does not change the symmetric form of the atomic motion equation; thus no asymmetric behaviors would appear in the atomic diffraction.

Like the interaction effects discussed in Ref. [17], in the quantized light fields, the atomic interactions also greatly modify the atomic coherence of the condensate. The atomic motion could be accelerated by a repulsive interaction and decelerated by an attractive interaction. Similarly, the repulsive and attractive interactions establish distinguishable equilibriums for the atomic occupations among different diffraction orders. From the results shown in Figs. 6 and 7, one finds that the repulsive interaction tends to make the $n = 0, \pm 1$ diffraction orders become equally occupied, while for the attractive interaction, the atomic numbers on $n = 0, \pm 1$ orders would deviate from each other.

VI. SUMMARY

In this paper, we investigate the atomic diffraction in quantized light fields. The quantum nature of the light fields induces exotic phenomena in the atomic scattering processes. For light fields in number states, the atomic transitions among different discrete momentum modes depend on both the transition order and the initial photon-number distribution. The quasiperiods of the atomic momentum oscillations are implicitly modified, and the atomic diffraction is no longer symmetric. However, for light fields in coherent states, the atomic diffraction is still symmetric; this verifies the coherent-state approximation of the light fields, which is frequently utilized in the cavity quantum electrodynamics. Situations of other kind of states of quantized light fields, such as arbitrary linear combinations of number states, can also be investigated using the methods introduced in this paper. Additionally, with the presence of the atomic interactions, the full-amplitude momentum oscillations of the BEC system would be dampened. The results obtained in this paper can be extended to other light-atom-interaction systems for enabling new discoveries in atom optics.

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