Effect on cavity optomechanics of the interaction between a cavity field and a one-dimensional interacting bosonic gas

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We investigate optomechanical coupling between one-dimensional interacting bosons and the electromagnetic field in a high-finesse optical cavity. We show that by tuning interatomic interactions, one can realize effective optomechanics with mechanical resonators ranging from side-mode excitations of a Bose-Einstein condensate (BEC) to particle-hole excitations of a Tonks-Girardeau (TG) gas. We propose that this unique feature can be formulated to detect the BEC-TG gas crossover and measure the sine-Gordon transition continuously and nondestructively.

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Experimental achievements in manipulating the strong coupling between ultracold atoms and the electromagnetic field in an optical cavity have triggered many exciting advances in cavity quantum electrodynamics (QED) [1–8]. One of the remarkable achievements is the implementation of cavity optomechanics with cold atoms [6,7] or a Bose-Einstein condensate (BEC) [8], which is of great importance for both technical applications, ranging from optical communication to quantum computation [9], and conceptional exploration of classic-quantum boundaries [10].

In this paper, we investigate the optomechanical coupling between a one-dimensional (1D) interacting bosonic gas and a cavity field. Recent works have neglected interatomic interactions or considered merely the weakly interacting region, where mean-field Bogoliubov theory is valid [8,11]. In this case, the 1D bosonic gas forms a BEC (or quasi-condensate) [12], and the side-mode excitations of the condensate play the role of mechanical resonator with the bare frequency $\omega_M^0 = 4\hbar k^2/M$ [8], where $k = 2\pi/\lambda_c$ is the wave vector of the cavity mode. However, when interatomic interactions are added into the system, the situation changes dramatically. The strong interatomic interactions transform the ground state of the condensate to a Luttinger liquid (LL). Remarkably in a strongly interacting situation, the 1D bosons—known as a Tonks-Girardeau (TG) gas [13–16]—exhibit completely different behavior, like ideal fermions. It is therefore important to explore the interatomic-interaction effects on cavity optomechanics, where the quantum fluctuations of 1D bosons are very strong.

In this work, we first employ the quantum hydrodynamical approach to derive an effective model of the cavity QED with 1D interacting bosons. We show that effective optomechanics can be realized in intermediate and strongly interacting regions. The corresponding optomechanical coupling is determined by low-energy excitations or interatomic interactions of the bosons. Therefore, by probing the cavity oscillations or the noise spectra versus the interatomic interactions, one can determine the quantum phases of the 1D interacting gas and detect the BEC-TG gas crossover, a fascinating phenomenon of the system [17]. Furthermore, we propose that one could also measure the sine-Gordon transition, which has stimulated considerable interest [18,19], conveniently with nondestructive measurements [20,21].

The system under investigation is schematically depicted in Fig. 1(a), where N is the ultracold bosonic atoms of mass M with a resonant frequency ω_a confined in a 1D trap inside an optical cavity with length L. The cavity mode of frequency ω_c is driven by a pump laser of frequency ω_p at rate η, and κ is the decay rate of the cavity field. Following Ref. [22], we adiabatically eliminate the internally excited state of the atoms, as justified by the large detuning between the atomic resonance and pump frequency. Then, by using the dipole and rotating-wave approximations, one arrives at the following Hamiltonian of the atomic part:

$$\hat{H}_a = \frac{\hbar^2}{2M}\int_0^L dx\,\partial_x\hat{\Psi}^\dagger(x)\partial_x\hat{\Psi}(x) + \int_0^L dx\,\hat{V}(x)\hat{\rho}(x) + \frac{1}{2}\int_0^L dx dx'\,\hat{\rho}(x)U(x-x')\hat{\rho}(x'). \quad (1)$$

Here, $\hat{\Psi}(x)$ is the bosonic field operator, $\hat{\rho}(x) = \hat{\Psi}^\dagger(x)\hat{\Psi}(x)$ is the atomic density operator, and $\hat{V}(x) = \hbar U_0\cos^2(kx)\hat{c}^\dagger\hat{c}$ is the dynamical periodic potential, with $\hat{c}$ the annihilation operator of a cavity photon and $U_0 = g_0^2/(\omega_p - \omega_a)$ the potential depth. The interatomic interactions are given by contact pseudo-potentials $U(x-x') = g_{1d}\delta(x-x')$, where $g_{1d} = \frac{2\hbar^2 a_s}{(1-\mathcal{C}a_s/\sqrt{2}l_\perp)Ml_\perp^2}$ is the effective 1D coupling strength with a_s the three-dimensional scattering length, $\mathcal{C} = 1.0325$, and $l_\perp = \sqrt{\hbar/M\omega_\perp}$ the transverse oscillator length.

We start by considering the general situation with arbitrary interatomic interactions and derive an effective model of the system by using the quantum hydrodynamical approach [23], which is a well-defined low-energy theory. We work in the low-photon-number limit, where the dynamical periodic potential $\hat{V}(x)$ is negligible. By introducing two new fields $\hat{\phi}(x)$ and $\hat{\theta}(x)$, which describe the collective fluctuations of the

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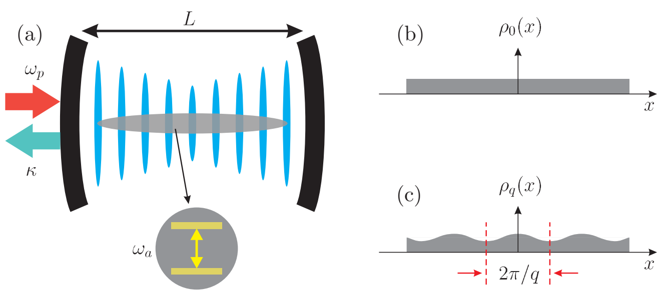

FIG. 1. (Color online) Experimental setup and schematic density distribution. (a) N bosonic atoms with resonant frequency ω_a are confined in an effectively 1D trap inside an optical cavity of length L. The cavity mode is driven by a pump laser of frequency ω_p and the decay rate is κ. (b) Ground-state atomic density distribution $\rho_0(x)$ in the absence of a cavity mode. (c) Distribution of density fluctuation $\rho_q(x)$ with wave vector $q = \pm 2k$, which is scattered by the periodic potential of the cavity mode.

phase and density, respectively, and satisfy the commutation relation $[\hat{\phi}(x), \frac{1}{\pi}\partial_{x'}\hat{\theta}(x')] = i\delta(x - x')$, we can express the bosonic field operator $\hat{\Psi}(x)$ as

$$\hat{\Psi}(x) \sim \left[\rho_0 - \frac{1}{\pi}\partial_x\hat{\theta}(x)\right]^{1/2} \left\{\sum_{m=-\infty}^{+\infty} e^{2mi\hat{\theta}(x)} e^{i\hat{\phi}(x)}\right\} \tag{2}$$

and the corresponding density operator as

$$\hat{\rho}(x) = \left[\rho_0 - \frac{1}{\pi}\partial_x\hat{\theta}(x)\right] \sum_{m=-\infty}^{+\infty} e^{2im[\hat{\theta}(x) - \pi\rho_0 x]}. \tag{3}$$

Here, $\rho_0 = N/L$ is the homogeneous ground-state atomic density.

In the following, we first consider the long-wavelength approximation, i.e., $\lambda_c \gg 1/\rho_0$, where we can only keep the $m = 0$ term in Eqs. (2) and (3). In this limit, the system can be expressed by a hydrodynamical description, with the weak dynamical periodic potential coupled to the slow part of the density operator $\hat{\rho}(x)$. Then, the corresponding low-energy effective Hamiltonian of the atomic part reads

$$\hat{H}'_a = \int_0^L dx \left(\frac{\hbar\upsilon_s}{2\pi}\left\{K[\partial_x\hat{\phi}(x)]^2 + \frac{1}{K}[\partial_x\hat{\theta}(x) - \pi\rho_0]^2\right\} - \frac{\hat{V}(x)}{\pi}\partial_x\hat{\theta}(x)\right). \tag{4}$$

Here, K is the dimensionless parameter, and υ_s is the sound velocity. They both depend on a single dimensionless interacting parameter $\gamma = Mg_{1d}/\hbar^2\rho_0$, with $\upsilon_s K \equiv \upsilon_F = \hbar\pi\rho_0/M$ fixed by Galilean invariance (also see Fig. 3 and the detailed discussion thereafter). We note that the Hamiltonian (4) describes a LL coupled to a weak periodic potential, which is dynamically dependent on the atomic state and determined self-consistently.

We further transform the Hamiltonian (4) to a momentum representation and then implement the standard bosonization procedure by introducing the bosonic creation operator $\hat{b}_q^\dagger = \sqrt{\frac{2\pi}{|q|L}}\hat{\rho}_q$. We can arrive at the following effective Hamiltonian of the coupled system:

$$\hat{H}_{\rm eff} = \sum_{q=\pm 2k} \hbar\omega_q \hat{b}_q^\dagger \hat{b}_q + \hbar g \sum_{q=\pm 2k} (\hat{b}_q^\dagger + \hat{b}_q)\hat{c}^\dagger\hat{c} + \hbar\Delta\hat{c}^\dagger\hat{c} + i\hbar\eta(\hat{c}^\dagger - \hat{c}). \tag{5}$$

Here the first term describes the long-wavelength density fluctuations of the 1D interacting gas with $\omega_q = \upsilon_s|q|$ for $|q| \ll \rho_0^{-1}$. The second term is the coupling between the corresponding density fluctuations and the cavity field with $g = \frac{U_0}{4}\sqrt{\frac{kL}{\pi}}$. In the sums, we assume that only the $q = \pm 2k$ modes are coupled to the cavity, which is justified by the low-photon-number limit. $\Delta = \omega_c - \omega_p + U_0 N/2$ is the effective cavity detuning.

The effective Hamiltonian (5) actually describes the optomechanical coupling between a mechanical oscillator (with frequency $\omega_M = \omega_{\pm 2k} = 2k\upsilon_s$) and the radiation pressure force of a cavity field. To see this, we introduce the quadratures of the bosonic excitations $\hat{X}_M = \sum_{q=\pm 2k}(\hat{b}_q^\dagger + \hat{b}_q)/\sqrt{2}$, and we derive the following Heisenberg-Langevin equations:

$$\frac{d^2\hat{X}_M}{dt^2} + \omega_M^2\hat{X}_M = -2\sqrt{2}g\omega_M\hat{c}^\dagger\hat{c}, \tag{6}$$

$$\frac{d\hat{c}}{dt} = -i\Delta_{\rm eff}\,\hat{c} + \eta - \kappa\hat{c} + \sqrt{2\kappa}\hat{c}_{\rm in}, \tag{7}$$

with the resonance frequency $\Delta_{\rm eff} = \Delta + \sqrt{2}g\hat{X}_M$ and the noise term $\sqrt{2\kappa}\hat{c}_{\rm in}$. Here, the low-energy, long-wavelength density fluctuation (phonon) plays the role of a mechanical resonator.

Now, we investigate optomechanical coupling governed by the set of coupled equations (6) and (7). One of the characteristic phenomenon of cavity optomechanics is the bistable behavior of the stationary solutions, which we derive as $\bar{X}_M = -2\sqrt{2}g|\alpha|^2/\omega_M$, and mean photon number $|\alpha|^2 = \eta^2/[\kappa^2 + (\Delta - 4g^2\omega_M^{-1}|\alpha|^2)^2]$. In Fig. 2(a), we give the bistability for three typical oscillators with frequency ω_M, which corresponds to different quantum phases of the system. A linear stability analysis shows that the middle branch (dashed line) is unstable, while the upper and lower branches are stable. Here, typical experimental parameters are used: $L \sim 100\,\mu$m, $\lambda_c = 780$ nm, $N \simeq 5000$ ^{87}Rb atoms, with $1/\rho_0 \simeq 20$ nm $\ll \lambda_c$ satisfying the wavelength approximation and $\kappa = 2\pi \times 1$ MHz, $U_0 = 2\pi \times 20$ kHz with the Rabi frequency $g_0 = 2\pi \times 10.9$ MHz and the pump-atom detuning $\omega_p - \omega_a = 2\pi \times 32$ GHz.

To discuss the dynamics of optomechanics, we have taken into account the lossy and driven cavity, where quantum jumps in the cavity photon number can lead to a strong entanglement between the cavity photon number and the bosonic wave function. This creates a displacement noise spectrum of the mechanical oscillator $S_{X_M}(\omega) = 2\kappa(4g\alpha\omega_M)^2[\kappa^2 + (\tilde{\Delta} + \omega)^2]/|d(\omega)|^2$ and the corresponding measurable noise spectrum of the cavity field quadrature $\hat{X}_c = (\hat{c}^\dagger + \hat{c})/\sqrt{2}$ [24], which gives $S_{X_c}(\omega) = \{(2g\alpha\tilde{\Delta})^2 S_{X_M}(\omega) + 2\kappa[\kappa^2 + (\tilde{\Delta} + \omega)^2\}/|d(\omega)|^2$ [see Fig. 2(c)]. Here, $d(\omega) = (\omega^2 - \omega_M^2)[(\kappa - i\omega)^2 + \tilde{\Delta}^2] + 2\omega_M\tilde{\Delta}(2g\alpha)^2$, and $\tilde{\Delta} = \Delta - (2g\alpha)^2/\omega_M$.

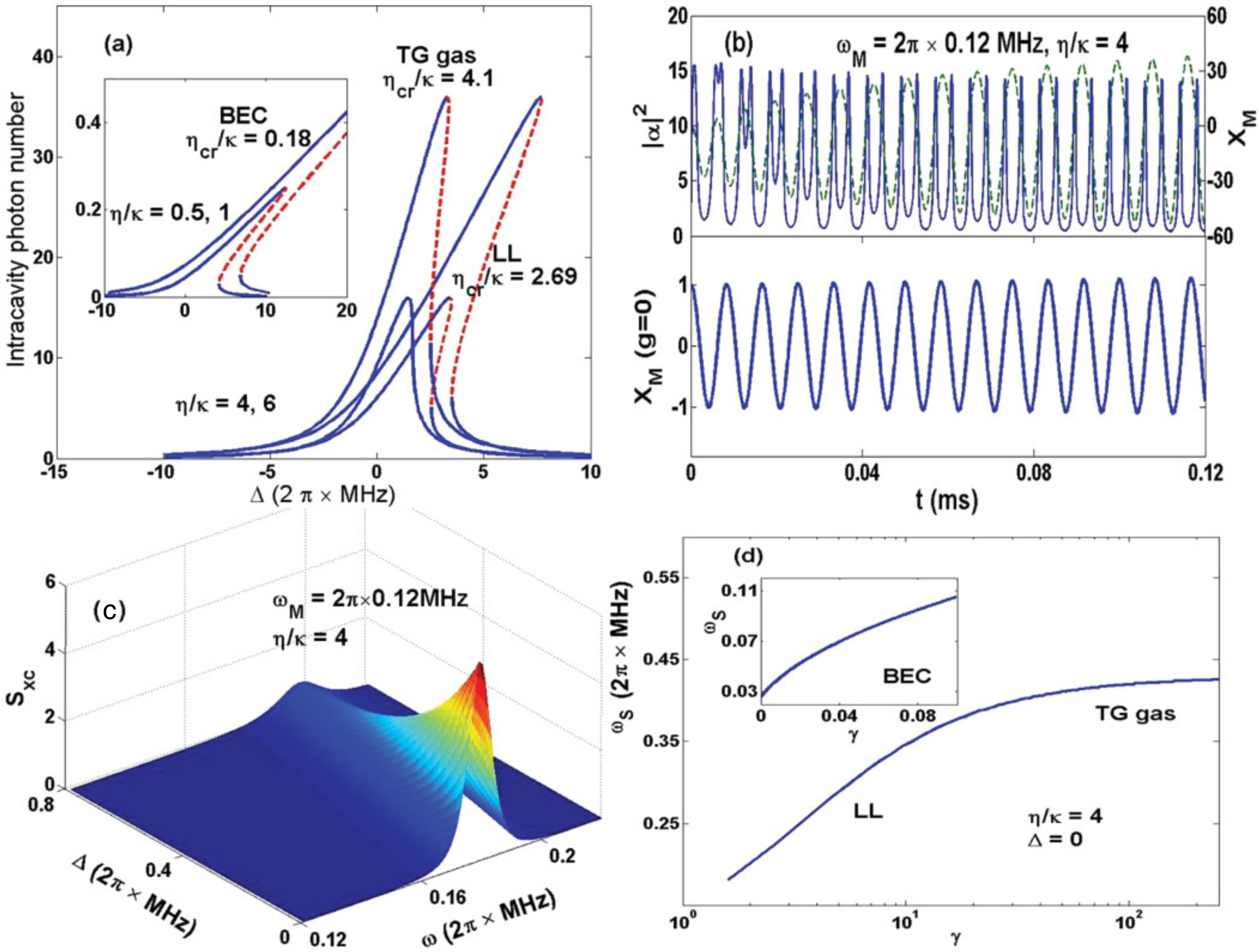

FIG. 2. (Color online) Steady-state and dynamical behavior of the optomechanical coupling. (a) Mean cavity photon number vs Δ for $\omega_M = 2\pi \times [0.02\ \text{(BEC)}, 0.12\ \text{(LL)}, 0.28\ \text{(TG gas)}]$ MHz. Here $\eta_{\rm cr}$ is the bistability threshold. (b) Cavity photon number $|\alpha|^2$, coupled oscillator X_M (dashed line), and free oscillator $X_M(g=0)$ for $\Delta = 0$. (c) Noise spectrum S_{X_c} vs ω and Δ. (d) Center frequency ω_S ($\times 2\pi$ MHz) of S_{X_c} vs γ. The inset shows the results for the small-γ region.

We then explore interatomic-interaction dependence on effective optomechanics. First we note that, whether the system is in weakly or strongly interacting regions, one can realize the effective mechanical oscillator in the whole region. This implies that the 1D bosonic gas is in fact in a universal class, which can be well described by low-energy hydrodynamical theory. However, the mechanical oscillator frequency ω_M is fully dependent on the interatomic-interaction parameter γ with $\omega_M = 2k\upsilon_s$. Here, the sound velocity is given by $\upsilon_s = \sqrt{(\rho_0/M)\,\partial^2 E/\partial N^2}$, and $E = \sum_l \hbar^2 k_l^2/2M$ is the ground-state energy of the 1D bosonic gas with k_l determined by the following Bethe ansatz equations [14]:

$$k_l L = 2\pi I_l - \sum_{m=1}^{N} \tan^{-1}\left(\frac{k_l - k_m}{\gamma\rho_0}\right), \quad (8)$$

where $I_l \in \{-(N-1)/2, \ldots, (N-1)/2\}$ are the set of integers. We solve Eq. (8), and in Fig. 3 we show the numerical results of ω_M versus γ. Experimentally, when photons enter the cavity, the light field and the bosonic excitations are coupled nonlinearly, and the eigenfrequency of Eqs. (6) and (7) will modify ω_M. Nevertheless, the optomechanical coupling is fully determined by the frequency ω_M of bosonic excitations and thereby dependent on γ of the 1D gas. Accordingly, by measuring the cavity field oscillations or detecting the noise spectrum, one can determine the continuous BEC-TG gas crossover, which is an intriguing result of the system.

In the weakly interacting region, we can use the Bogoliubov approximation to derive $\omega_M = 2k\upsilon_F\sqrt{\gamma - \gamma^{3/2}/(2\pi)}/\pi$ for $\gamma \leqslant 10$ (the dashed line of Fig. 3). However, we note that ω_M vanishes as $\gamma \to 0$, which contradicts recent experimental results [8]. In fact, because the bosons begin to quasi-condense for $\gamma \ll 1$, the dominant contribution of the density fluctuation $\hat{\rho}_{\pm 2k} = \hat{\Psi}^\dagger_{\pm 2k}\hat{\Psi}_{q=0} + \sum_{q\neq 0}\hat{\Psi}^\dagger_{\pm 2k+q}\hat{\Psi}_q$ comes from the quasiparticle excitations from the macroscopic occupied $q = 0$ ground state. The energy of a quasiparticle is then determined by the Bogoliubov excitation spectrum $\omega_M = \sqrt{\epsilon_{\pm 2k}(\epsilon_{\pm 2k} + 2g_{1d}\rho_0)}/\hbar$ with $\epsilon_{\pm 2k} = 2\hbar^2 k^2/M$ (the dotted line of Fig. 3). In this case, the mechanical oscillators are the side-mode excitations of a BEC, and in the limit of $\gamma = 0$,

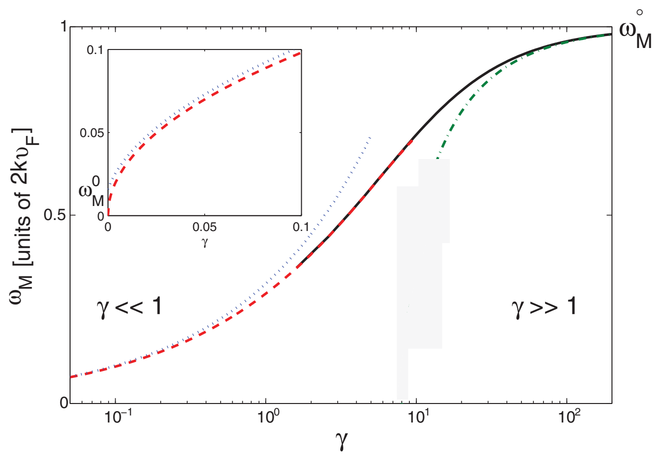

FIG. 3. (Color online) Mechanical oscillator frequency ω_M (in units of $2k\upsilon_F$) vs γ. The solid line is the numerical Bethe ansatz result. The dashed and the dash-dotted lines are the asymptotic results in the weakly and strongly interacting regions, respectively. In the weakly interacting region, we also give the Bogoliubov excitation spectrum (dotted line).

we get the bare oscillator frequency $\omega_M^0 = 4\hbar k^2/M$ (inset of Fig. 3). Accordingly, the coupling between the oscillator and a cavity field should be replaced by $g = U_0/2\sqrt{N/2}$ in Eqs. (6) and (7), which is enhanced by the condensation. We give the stationary bistability in the inset of Fig. 2(a), and then numerically integrate the coupled equations for $\Delta = 0$ by switching on $\eta/\kappa = 4$ at $t = 0$. We find that the cavity field oscillates regularly (not shown), but the frequency of the cavity field has a large shift of ω_M, which agrees well with Ref. [8].

When γ is further increased above unity, the collective density excitations become dominant and the 1D gas crosses to a LL [17]. Numerical integration of Eqs. (6) and (7) shows that both $|\alpha|^2$ and X_M exhibit well-defined oscillations, and the cavity field is excited resonantly at $X_M = 0$ [Fig. 2(b)]. Yet we note that, different from the BEC phase where g is collectively enhanced, the optomechanical coupling becomes small. In this case, a linear stability analysis shows that the eigenfrequency of the set of coupled equations is nearly the same as for the free oscillator ω_M [for example, see Fig. 2(b)]. This is a characteristic phenomenon of the region. Experimentally, by increasing the interatomic interactions, if the oscillation frequency of the cavity field follows the solid line of Fig. 3, the 1D gas should be in the LL phase. We also calculate the noise spectrum S_{X_c} [Fig. 2(c)], where the center frequency ω_S of the spectrum has a shift of ω_M. In Fig. 2(d), we give ω_S for $\Delta = 0$ in the whole interacting region. We find that ω_S increases with γ and has a tendency to saturate above $\gamma \simeq 50$, which can be inspected in experiments.

For $\gamma \gg 1$, the strong interactions would prevent the bosons from occupying the same position. Especially when $\gamma = \infty$, the symmetric many-body wave function of bosons can be mapped to an antisymmetric fermionic wave function by $\Psi_B(x_1,\ldots,x_N) = A(x_1,\ldots,x_N)\Psi_F(x_1,\ldots,x_N)$ with $A(x_1,\ldots,x_N) = \prod_{1\leqslant j<k\leqslant N}\mathrm{sgn}(x_k - x_j)$ [13]. Hence, the Hamiltonian (1) can be rewritten in terms of the fermion field operators:

$$\hat{H}_F = \frac{\hbar^2}{2M}\int_0^L dx\,\partial_x\hat{\Psi}_F^\dagger(x)\partial_x\hat{\Psi}_F(x) + \int_0^L dx\,\hat{V}(x)\hat{\rho}_F(x), \tag{9}$$

which is exactly the model describing a free fermion gas subjected to the cavity periodic potential [25]. Then, the oscillator is formed by particle-hole excitations at the edges of $\pm k_F$ through Bose-Fermi mapping, and the mechanical frequency is naturally related to the Fermi velocity: $\omega_M^\infty = 2k\upsilon_F = 2k\hbar\pi\rho_0/M$. For finite γ, we use the asymptotic expression to derive $\omega_M = 2k\upsilon_F[1 - 4/\gamma]$ (the dash-dotted line of Fig. 3). Experimentally, when the 1D gas becomes a TG gas, the oscillation frequency of the cavity field should follow this asymptotic expression and should approach ω_M^∞. Correspondingly, the center frequency ω_S will saturate at $2\pi\times 0.42$ MHz [Fig. 2(d)].

Let us now turn to the commensurate situation with $\lambda_c \sim 2/\rho_0$, where a new instability—the sine-Gordon transition—may appear in the strongly interacting 1D quantum gas [19]. Here, the superfluid ground state turns insulating in the presence of a weak commensurate periodic potential. It is now necessary to take account of the discrete nature of the boson with $m \neq 0$ terms in Eq. (3). This gives rise to a sine-Gordon-type perturbation [18] up to the leading term,

$$\hat{H}_{\text{s-G}} = \frac{1}{2}\hbar U_0\hat{c}^\dagger\hat{c}\rho_0\int_0^L dx\cos[2\hat{\theta}(x) + Qx], \tag{10}$$

where $Q = 2\pi(\rho_0 - k/\pi)$, which vanishes at commensurability. In the small-photon-number limit, it was shown in [26] that this term is renormalization independent for $K > K_c = 2$, or equivalently $\gamma < \gamma_c = 3.5$, leaving the ground state a superfluid LL with the same linear excitation spectrum as in the long-wavelength approximation. Then, one expects a well-defined oscillation of the optomechanical coupling. While for $\gamma > \gamma_c$, $\hat{H}_{\text{s-G}}$ becomes relevant, the system transitions to an insulating Mott phase, where the bosonic excitations are forbidden owing to the energy cost, and the optomechanical oscillation vanishes correspondingly. Therefore, in experiments, we can easily use optomechanical dynamics across the critical point γ_c to detect the sine-Gordon transition.

Finally, we remark on several issues related to our work. First, we note that some literature on nonmeanfield theories of bosonic fields in cavity light fields already exists. For example, in Ref. [27], the authors studied the commensurate situation with large photon numbers, where the sine-Gordon transition eventually evolves into a Mott transition of a Bose-Hubbard type combined with a nonlinear cavity mode. In our work, we explore the non-mean-field effects on cavity optomechanical oscillations in the low-photon-number limit, which differs from existing works. Second, when the bosons are incommensurate with $Q \neq 0$, the system undergoes a commensurate-incommensurate-type transition [18], which we propose may also be probed in the cavity. Third, the mechanical frequency ω_M of the interacting gas is generally determined by the optical potential in classical and semiclassical regimes. In this paper, we focus on the small-photon-number limit (see Ref. [8] for the experimental conditions), where the influence of the cavity field on cold atoms can be treated perturbatively by standard linear response theory (LRT). In this situation, the energy dispersion of the relevant quasiparticles, which determines the mechanical frequency ω_M, is solely dependent on the intrinsic many-body atomic state. Beyond LRT, the oscillation frequency would be modified by the photon number and the atom-photon coupling strength g.

In summary, we demonstrate that one can realize effective optomechanics in the whole interacting regions of 1D bosonic gas. This offers an approach to detecting the BEC-TG gas crossover or the sine-Gordon transition by investigating optomechanical coupling. These proposals are of particular significance for exploring novel phenomena of cavity QED and ultracold atoms.

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