

Testing the equivalence between the canonical and Minkowski momentum of light with ultracold atoms

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We design an experimental system to test the equivalence between the canonical and Minkowski momentum of light, which can provide a judgment on the recent resolution of the century-old Abraham-Minkowski controversy by S. M. Barnett [Phys. Rev. Lett. **104**, 070401 (2010)]. By measuring the recoil momentum of ultracold rubidium atoms in a rubidium Bose-Einstein condensate after the electromagnetically induced absorption of a monochromatic laser pulse, the momentum of the pulse in the ultracold atoms can be obtained. If the equivalence is valid, the measured results will coincide with the theoretical values of the canonical momentum of the pulse. Otherwise, if the equivalence is invalid, the measured results will coincide with the theoretical values of the Minkowski momentum, which are significantly different from that of the canonical momentum. Our scheme is a method to test the equivalence between the canonical and Minkowski momentum of light. It can also be improved to distinguish between the Minkowski and Abraham momenta to contribute to the study of the Abraham-Minkowski controversy in the future.

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I. INTRODUCTION

The study of the momentum of light in optical media, or, more generally speaking, the study of the energy-momentum tensor of the refracted light, raised the century-old problem known as the Abraham-Minkowski controversy [1–5]. In 1908, Minkowski suggested that the momentum of light in optical media is $\int(\mathbf{D} \times \mathbf{B})dV$ [6], whose single-photon expectation value is $n\hbar\mathbf{k}_0$. In 1909, Abraham argued that this momentum should be $\int(\mathbf{E} \times \mathbf{H}/c^2)dV$ [7], with the single-photon expectation value being $\hbar\mathbf{k}_0/n$. Here, \mathbf{k}_0 is the wave vector of light in vacuum, and n is the refractive index of the optical medium. Over the last century, several experiments were carried out in order to solve this controversy [8–11], even though the precision of their measurements was restricted by the experimental techniques at that time. In this century, a recent experiment measuring the recoil momentum of a Bose-Einstein condensate supports Minkowski's opinion [12], while another experiment measuring the recoil momentum of silica filaments favors Abraham's opinion [13] (with their result being questioned by several recent works [14–17]).

Recently, Barnett presented a resolution of the Abraham-Minkowski controversy [18], which is important progress in solving this long-standing problem. He demonstrated that the Abraham momentum is the kinetic momentum of light, whose single-photon expectation value is $\hbar\mathbf{k}_0/n_g$, with n_g being the group refractive index of the medium; the Minkowski momentum is the canonical momentum of light whose single-photon expectation value is $n_p\hbar\mathbf{k}_0$, with n_p being the phase refractive index of the medium. The unique total momentum \mathbf{P} of the light-optical-medium interacting system can be expressed as two different combinations [18–21],

$$\mathbf{P} = \mathbf{P}_k + \mathbf{P}_{\text{Abr}} = \mathbf{P}_c + \mathbf{P}_{\text{Min}}, \quad (1)$$

where \mathbf{P}_k and \mathbf{P}_c are the kinetic and canonical momenta of the optical medium and \mathbf{P}_{Abr} and \mathbf{P}_{Min} are the Abraham and Minkowski momenta of light [18], respectively.

The key point of Barnett's resolution is the equivalence between the canonical and Minkowski momenta of light in optical media. Before his work, Garrison and Chiao quantized the electromagnetic field in dispersive media [22] and gave the single-photon expectation values of its canonical, Minkowski, and Abraham forms:

$$\begin{aligned} \mathbf{P}_{\text{can}} &= n_p(\omega)\hbar\mathbf{k}_0, \\ \mathbf{P}_{\text{Min}} &= \frac{n_p^2(\omega)}{n_g(\omega)}\hbar\mathbf{k}_0, \\ \mathbf{P}_{\text{Abr}} &= \frac{1}{n_g(\omega)}\hbar\mathbf{k}_0, \end{aligned} \quad (2)$$

where ω is the angular frequency of light. Barnett suggests that the Minkowski momentum of light must include all the polariton branches and each branch i contributes to the commutation relation with the value $n_p(\omega_i)/n_g(\omega_i)$. Then there is a velocity summation rule [18],

$$\sum_i \frac{n_p(\omega_i)}{n_g(\omega_i)} = \sum_i \frac{v_g(\omega_i)}{v_p(\omega_i)} = 1, \quad (3)$$

which leads to the summation of the single-photon expectation values of the Minkowski momentum among all bands being equivalent to the canonical momentum,

$$\sum_i \frac{n_p^2(\omega_i)}{n_g(\omega_i)}\hbar\mathbf{k}_0 \approx \sum_i \frac{n_p(\omega_i)}{n_g(\omega_i)}\mathbf{P}_{\text{can}} = \mathbf{P}_{\text{can}}. \quad (4)$$

Here ω_i is the solution of the dispersion relation $\omega = c\mathbf{k}_0(\omega)/n_p(\omega)$ for polariton branch i [18], $v_p(\omega_i) = c/n_p(\omega_i)$ is its phase velocity, and $v_g(\omega_i) = c/n_g(\omega_i)$ is its group velocity.

Barnett also argues that the single-photon expectation value of the Minkowski momentum $n_p^2(\omega)\hbar\mathbf{k}_0/n_g(\omega)$ cannot be detected because the *narrow-band approximation* is invalid in optical media. Then all the polariton modes, whether excited or not, must be included in the summation in Eq. (3). However, in Refs. [22,23], the Minkowski momentum is

derived with the narrow-band approximation (without the summation of all polariton modes), and its single-photon expectation value in Eq. (2) is different from the one of the canonical momentum. Therefore, the equivalence between the Minkowski and canonical momenta needs to be experimentally tested in order to judge Barnett's resolution, which is important for the study of the Abraham-Minkowski controversy.

In this paper, we design an experiment to measure the single-photon expectation value of the momentum of a monochromatic laser pulse in dispersive ultracold atoms, which can test well the equivalence between the canonical and Minkowski momenta of light. In Sec. II, we present the principle characteristics and scheme of the experiment and describe how to measure the recoil momentum curve from the recoil pattern of ultracold atoms. In Sec. III, both the canonical and Minkowski recoil momenta of the ultracold atoms with the electromagnetically induced absorption (EIA) of the monochromatic pulse are calculated. We show that if the equivalence between the canonical and Minkowski momenta of light is valid, the measured recoil momentum curve will coincide with the curve of the recoil canonical momentum; if the equivalence is invalid, the measured recoil momentum curve will follow the curve of the single-photon values of the Minkowski momentum, which is quite different from that of the canonical momentum. These differences can be detected well in our proposed experiment. In Sec. IV, we also show the possibility of observing the differences between the Minkowski and Abraham momenta of the laser pulse with the same setup. Section V is the summary and conclusion.

II. SCHEME OF THE EXPERIMENT

Many recent theoretical [24–26] and experimental [12] works suggest that the momentum of a photon in an atomic medium should take the Minkowski value $n_p \hbar \mathbf{k}_0$. Thus a stationary atom in the medium which absorbs the photon from the pulsed pump light will get a recoil momentum of $n_p \hbar \mathbf{k}_0$. However, according to Garrison and Chiao [22], the above momentum is actually the canonical momentum of light, and the single-photon expectation value of the Minkowski momentum is indeed $n_p^2 \hbar \mathbf{k}_0 / n_g$ after the quantization of the Minkowski momentum tensor in dispersive optical media.

Barnett argues that these two momenta are equivalent due to the summation rule in Eq. (3) and that many experimental attempts that try to measure the value of $n_p^2 \hbar \mathbf{k}_0 / n_g$ would finally obtain the value of $n_p \hbar \mathbf{k}_0$. For nondispersive atomic media, these two momenta are always the same because the phase refractive index n_p is equal to the group refractive index n_g in such media, which leads to $n_p \hbar \mathbf{k}_0 = n_p^2 \hbar \mathbf{k}_0 / n_g$. However, for dispersive atomic media, there is a dispersion relation,

$$n_g(\omega) = n_p(\omega) + \omega \frac{dn_p(\omega)}{d\omega}, \quad (5)$$

which makes n_p significantly different from n_g when $|\omega \frac{dn_p(\omega)}{d\omega}| \gg n_p(\omega)$. According to the summation rule in Eq. (3), no matter how different n_p and n_g are from each other, the measured photon's momentum would still take the canonical form $n_p \hbar \mathbf{k}_0$. However, in atomic media, only one polariton mode is excited by a monochromatic light

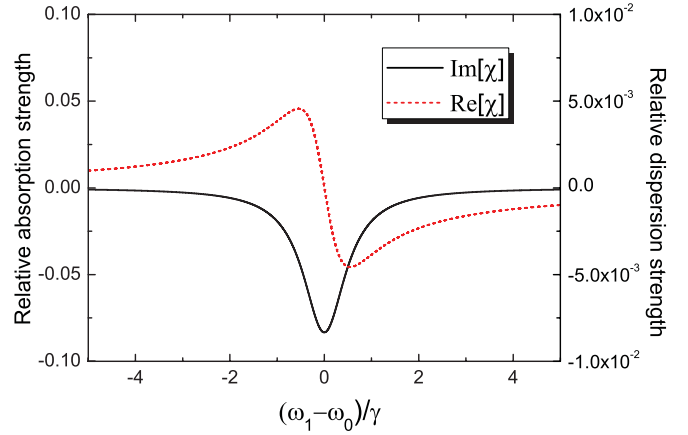


FIG. 1. (Color online) Absorption curves (solid black line) and dispersion curves (dashed red line) of the EIA obtained from the Kramers-Kronig relation, $\text{Im}[\chi]$ and $\text{Re}[\chi]$, respectively. Here ω_0 is the frequency of the strong pump light, and ω_1 is the frequency of the weak probe light. The EIA happens when ω_1 is close to ω_0 , and the absorption peak is just at $\omega_1 = \omega_0$. The width of the anomalous dispersion range is equal to γ (full width at half maximum) of the absorption profile.

(with a unique dispersion relation). If the summation rule in Eq. (3) does not hold for unexcited modes (which means that the narrow-band approximation is valid here), the measured monochromatic photon's momentum in atomic media may take the Minkowski form $n_p^2 \hbar \mathbf{k}_0 / n_g$, which would deviate greatly from the values of the canonical form $n_p \hbar \mathbf{k}_0$. Therefore, in order to test the equivalence between the canonical and Minkowski momenta, an atomic medium with a unique dispersion curve for monochromatic light would be very suitable.

In the experiment of Campbell *et al.* [12], the dispersion curve corresponds to the absorption signal of the ^{87}Rb $5^2S_{1/2}, F=1 \rightarrow 5^2P_{3/2}, F'=1$ transition, whose natural width is 6.056 MHz. Thus the peak-to-peak width of the dispersion curve is also 6.056 MHz. This value is wide, and the measured points in their experiment are all far away from the center of the dispersion curve. Thus the values of $\omega \frac{dn_p(\omega)}{d\omega}$ for their measured points would all satisfy

$$\omega \frac{dn_p(\omega)}{d\omega} \ll n_p(\omega). \quad (6)$$

This makes their measurement inadequate to distinguish between the values of $n_p(\omega) \hbar \mathbf{k}_0$ and $n_p(\omega)^2 \hbar \mathbf{k}_0 / n_g(\omega)$ from the recoil patterns of the ultracold atoms.

In our scheme, we choose the EIA [27] process to let the ultracold atoms [typically, Bose-Einstein condensate (BEC)] get recoil momentum from the refracted photons. Figure 1 shows the absorption and dispersion curves of the EIA, where the EIA signal is a sharp absorption signal (typically < 1 MHz) with a width much narrower than the natural width of the ^{87}Rb atoms. Since the imaginary part of the electric susceptibility χ is proportional to the absorption strength and the real part of χ is proportional to $(n_p^2 - 1)$ [28], the EIA signal has a strong anomalous dispersion range ($d[n_p(\omega)]/d\omega < 0$) due to the Kramers-Kronig relation. This can even lead to $d[n_p(\omega)]/d\omega \approx -6 \times 10^{-11}/\text{Hz}$ [29].

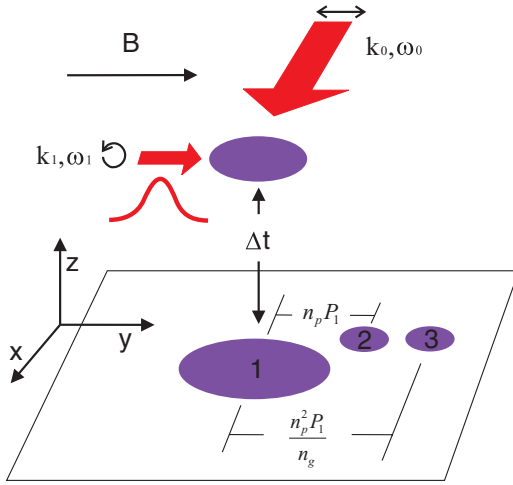


FIG. 2. (Color online) Experimental setup measuring the recoil momentum of ultracold ^{87}Rb atoms by EIA. The strong linearly polarized pump beam (k_0, ω_0) propagates along the x direction, and the circularly polarized weak probe beam (k_1, ω_1) propagates along the y direction. At the bottom, the pattern of the atoms' free diffusion (delayed by Δt after EIA) can be detected by a CCD camera. Region 1 is the pattern of the atoms that do not participate in the EIA. Region 2 is the pattern of the atoms that obtain a recoil momentum equal to the canonical momentum of the pulse, and region 3 is the pattern of the atoms that obtain a recoil momentum equal to the Minkowski momentum of the pulse. $\mathbf{P}_1 = \hbar \mathbf{k}_1$ is the free momentum of the probe light, while n_p and n_g are the phase and group refractive indices of the atoms, respectively.

Figure 2 shows the scheme of our design. The ^{87}Rb BECs are originally trapped in a magnetic or a far-detuned optical trap. The atomic density can reach up to $10^{15}/\text{cm}^3$ whenever the volume of the ^{87}Rb is compressed by increasing the depth of the magnetic or optical trap. The pump and probe laser beams for EIA propagate below the trap region. The distance from the bottom of the trap to the edge of the beams should be less than 1 mm. The pump laser beams (\mathbf{k}_0, ω_0) are continuous waves with a narrow width (~ 100 kHz). The probe beam (\mathbf{k}_1, ω_1) is a ~ 10 - μs pulse. The light intensity of the pump beam shall be set at around 50 mW/cm 2 , and the peak intensity of the probe pulse is weaker than 0.1 mW/cm 2 . The two beams are perpendicular to each other to make sure that each beam contributes to the atomic recoil momentum independently in the x and y directions. The atoms get recoil momentum in the y direction only by absorbing refracted photons from the probe beam, and they recoil in the x direction only by absorbing refracted photons from the pump beam. The spots of the two beams should be larger than 0.5 mm 2 to ensure that the atoms stay for a long enough time (> 10 ms) for the EIA process. The detunings of the two beams should be larger than 5Γ to avoid the $D2$ line resonant absorption. With an external magnetic field applied in the y direction, the pump beam is linearly polarized, and the probe pulse is circularly polarized in the y direction. Such a configuration can enhance the EIA process because the transition types induced by pump and probe lights are not same [30].

At $t = 0$, the trap is switched off, and the ^{87}Rb atoms fall into the intersecting region of the pump and probe beams. The

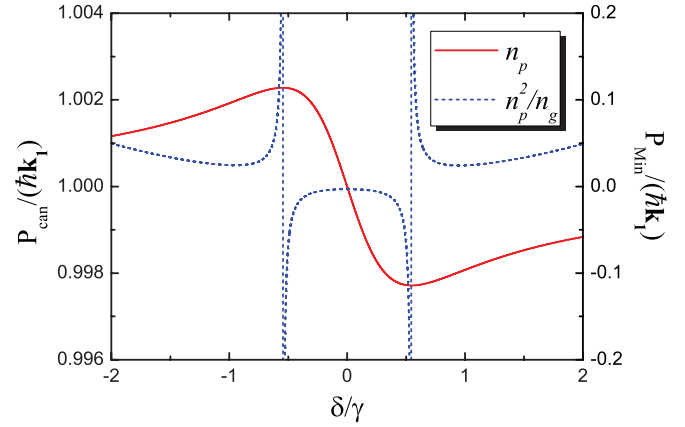


FIG. 3. (Color online) The comparison between canonical and Minkowski recoil momenta of cold ^{87}Rb atoms. The solid red line is the recoil momentum which is equal to the canonical momentum of the probe light, and the dashed blue line is the recoil momentum that is equal to the Minkowski momentum of the probe light. The width of the EIA signal γ is set as 1.0 MHz, and the strength of the signal is set as half of the ^{87}Rb $F = 2 \rightarrow F' = 3$ resonant absorption strength. Here $\delta = \omega_1 - \omega_0$ is the detuning between the pump and probe lights. At the center of the n_p curve, $n_p = 1$, while n_p^2/n_g is close to zero. Near the two peaks of the n_p curve, the n_p^2/n_g curve has a clear shape transition between large positive and negative values.

time sequence should be controlled very precisely to make the probe pulse encounter the ^{87}Rb atoms at that moment. After $\Delta t = 60$ ms, the image of the free diffusion pattern of the ultracold ^{87}Rb atoms will demonstrate that some ^{87}Rb atoms acquire a recoil momentum from the probe pulse along the y axis. According to the previous experiments, such as electromagnetically induced transparency (EIT) in BECs [31] and two-photon recoil of BECs [12], an optimized number of the ultracold atoms in the BEC of our designed experiment is 5.0×10^6 , and an optimized peak density of the BEC is 1.5×10^{15} cm $^{-3}$, which ensures the visibility and resolution of the recoil patterns of the BEC by EIA.

III. TESTING THE EQUIVALENCE BETWEEN THE CANONICAL AND MINKOWSKI MOMENTA

When the patterns of the recoiled ultracold ^{87}Rb atoms are obtained, the recoil momentum of the atoms can be measured from them. Then the equivalence between the canonical and Minkowski momenta of light can be directly tested. If the recoil momentum is equal to the canonical momentum $n_p \hbar \mathbf{k}_1$ of the probe laser pulse, the recoiled atoms will appear in region 2 in Fig. 2; if the recoil momentum is equal to the Minkowski momentum $n_p^2 \hbar \mathbf{k}_1 / n_g$, the recoiled atoms will appear in region 3 in Fig. 2.

Figure 3 shows the calculated results of the canonical and Minkowski momenta of the probe pulse around the EIA frequency range. These data determine the positions of the patterns of regions 2 and 3 in Fig. 2. The width of the EIA signal is set to be 1.0 MHz, and its strength is set as half of the ^{87}Rb $F = 2 \rightarrow F' = 3$ absorption strength. These two

momenta can be calculated from

$$\begin{aligned} n_g(\omega_1) &= n_p(\omega_1) + \omega_1 \frac{dn_p(\omega_1)}{d\omega_1}, \\ \mathbf{P}_{\text{can}} &= n_p(\omega_1) \hbar \mathbf{k}_1, \\ \mathbf{P}_{\text{Min}} &= \frac{n_p^2(\omega_1)}{n_g(\omega_1)} \hbar \mathbf{k}_1, \end{aligned} \quad (7)$$

with ω_1 being the frequency of the probe pulse and ω_0 being the frequency of the pump beam.

From Fig. 3, we can see that the canonical and Minkowski momenta of light take significantly different values in the dispersion range. At the point $\omega_1 = \omega_0$, the phase refractive index $n_p(\omega_1) = 1$, and the canonical momentum is equal to $\hbar \mathbf{k}_1$. The group refractive index here satisfies $n_g(\omega_1) \gg 1$ because $\omega_1 \frac{dn_p(\omega_1)}{d\omega_1} \gg 1$; thus the Minkowski momentum $n_p^2(\omega_1)/n_g(\omega_1)$ is close to zero at this point. This would result in no recoiled ultracold atoms being observed in region 2. Another significant difference between the canonical and Minkowski momenta appears around the two points where $dn_p(\omega_1)/d\omega_1 = 0$. At these two points, the values of the Minkowski momentum $n_p^2(\omega_1) \hbar \mathbf{k}_1 / n_g(\omega_1)$ have sharp shifts between $n_p^2(\omega_1)/n_g(\omega_1) \ll -1$ and $n_p^2(\omega_1)/n_g(\omega_1) \gg 1$. This is because the group index $n_g(\omega_1)$ is very close to zero around these two points. Then the curve of the Minkowski momentum has much more significant shifts than that of the canonical momentum, which might make the distance between region 3 and region 1 much larger than that between region 2 and region 1 in Fig. 2.

The frequency width of the EIA signal is determined by the flying time of the atoms in the pump and probe beams [27,30] and the optical shifts of the energy levels of the atoms caused by the pump beam [32]. One question is whether the frequency width of the probe pulse is narrow enough to detect the sharp shifts of the Minkowski momentum around the two points $dn_p(\omega_1)/d\omega_1 = 0$. Our answer is yes. The external-cavity diode lasers (ECDL) usually have a typical width of 100 kHz after frequency locking and thus can provide enough frequency resolution to detect the Minkowski momentum in Fig. 3.

If the measured recoil momentum of the ultracold ^{87}Rb atom matches the curve of the canonical momentum in Fig. 3 (solid red line), then we can arrive at the conclusion that the summation rule in Eq. (3) works even if some polariton modes are not excited. This would provide evidence supporting the equivalence between the canonical and Minkowski momenta as well as Barnett's recent resolution of the Abraham-Minkowski controversy [18]. On the contrary, if the measured recoil momentum follows the curve of the Minkowski momentum in Fig. 3 (dashed blue line), the summation rule in Eq. (3) does not hold for ultracold atoms, and the equivalence between the canonical and Minkowski momenta of light in dispersive atomic media would be questionable.

IV. ABRAHAM MOMENTUM

We assume that the measured results in Sec. II coincide with the Minkowski values (dashed blue line in Fig. 3). Since the single-photon expectation value of the Abraham momentum $\hbar \mathbf{k}_0 / n_g$ is close to the one for the Minkowski momentum

$n_p^2 \hbar \mathbf{k}_0 / n_g$, we need to make sure that the measured results are the values of the Minkowski momentum rather than the Abraham momentum. From Eq. (2) we can see that the difference between the single-photon expectation values of the Minkowski and Abraham momenta is $\frac{n_p^2(\omega)-1}{n_g(\omega)} \hbar \mathbf{k}_0$, which is three orders of magnitude smaller than both the Minkowski and Abraham momenta of light in ultracold atoms. Therefore, it is not easy to distinguish them experimentally. Then the problem that remains is how to distinguish the Abraham momentum from the Minkowski momentum in our proposed experimental system.

Our study shows that, within the accuracy required to detect the curve of the canonical momentum (solid red line in Fig. 3), it is possible to distinguish the Abraham momentum from the Minkowski momentum. We take typical values for the parameters of the EIA signal in Fig. 1 and adopt them in Fig. 3. The frequency width γ of the EIA signal can be set as 1 MHz, and the strength is half the ^{87}Rb $F = 2 \rightarrow F' = 3$ resonant absorption strength (based on the observed EIA signal in dilute cold-atom gas [32]). With such parameters, the maximum value of $|n_p - 1|$ is of the order of 10^{-3} . In practice, the number of ^{87}Rb atoms in BEC can be more than 10^6 , and the density of BEC can reach $10^{15}/\text{cm}^3$; therefore the optical thickness of the BEC can lead to a much larger maximum value of $|n_p - 1|$ ($\sim 10^{-2}$). Therefore, it is possible to distinguish between the Minkowski and Abraham momenta in our proposed experimental setup with a high-resolution CCD. The two best points for distinguishing between the two momenta are at $dn_p(\omega)/d\omega = 0$, where the group refractive index $n_g(\omega)$ is equal to the phase index $n_p(\omega)$ according to Eq. (5). At these two points, the Minkowski momentum becomes $\mathbf{P}_{\text{Min}} = n_p \hbar \mathbf{k}_1$, which is just equal to the canonical momentum, while the Abraham momentum becomes $\mathbf{P}_{\text{Abr}} = \hbar \mathbf{k}_1 / n_p$. At the point where $dn_p(\omega)/d\omega = 0$ and $n_p > 1$, the Minkowski momentum $\mathbf{P}_{\text{Min}} > \hbar \mathbf{k}_1$ and the Abraham momentum $\mathbf{P}_{\text{Abr}} < \hbar \mathbf{k}_1$; at the other point, where $dn_p(\omega)/d\omega = 0$ and $n_p < 1$, the Minkowski momentum $\mathbf{P}_{\text{Min}} < \hbar \mathbf{k}_1$ and the Abraham momentum $\mathbf{P}_{\text{Abr}} > \hbar \mathbf{k}_1$. Such a difference can be detected from the recoil patterns of the ultracold atoms in Fig. 2 by a high-resolution CCD.

On the other hand, if the measured results of Sec. II coincide with the curve of the canonical momentum (solid red line in Fig. 3), it is not necessary and even not possible to distinguish between the Minkowski and Abraham momenta. Besides the summation rule for Minkowski momentum in Eq. (3), Liu and Zhang recently suggest that a similar sum rule for Abraham momentum also works [33]. It is

$$\sum_i \frac{1}{n_g(\omega_i) n_p(\omega_i)} = 1, \quad (8)$$

where the subscript i indicates the polariton modes. This sum rule is one of the two Huttner-Barnett velocity sum rules [34], while the other one is just Eq. (3). Then the summation of each polariton mode's Abraham momentum $\hbar \mathbf{k}_0 / n_g(\omega_i)$ is also equal to the canonical momentum $n_p(\omega) \hbar \mathbf{k}_0$ in [33]. In such a case, the canonical momentum, Minkowski momentum, and Abraham momentum would be equivalent. Thus, if the measured results support the canonical momentum, the results may also be interpreted as evidence for the

equivalence among the canonical, Minkowski, and Abraham momenta.

V. SUMMARY

We designed an experiment to detect the Minkowski momentum of a laser pulse by measuring the recoil momentum of ultracold atoms after electromagnetically induced absorption processes. Our results show that the canonical momentum $n_p(\omega_1)\hbar\mathbf{k}_1$ and the Minkowski momentum $\frac{n_p^2(\omega_1)}{n_g(\omega_1)}\hbar\mathbf{k}_1$ of the monochromatic laser pulse could induce quite different recoil patterns of the ^{87}Rb BEC. If the observed recoil patterns are consistent with the curve of the canonical momentum of the probe pulse, it would be evidence for the equivalence between the canonical and Minkowski momenta of light. On the other hand, if the recoil patterns follow the curve of the Minkowski momentum of the probe pulse, the equivalence between the canonical and the Minkowski momenta is violated. In the latter case, it is also possible to distinguish between the Minkowski momentum and the Abraham momentum of the laser pulse by increasing the density of the ^{87}Rb BEC.

Our method can be realized by any ^{87}Rb BEC experimental system where the time-of-flight pattern of the recoiled ultracold ^{87}Rb atoms can be detected. Since Barnett's resolution of the Abraham-Minkowski controversy is important progress in the study of the Abraham-Minkowski controversy, we hope that future experiments invoked by our method can give a clear and convincing test of the equivalence between the canonical and Minkowski momenta of light. In addition, we hope that such an experimental system can detect the difference between the Minkowski and Abraham momenta of light, which is of definite importance for studying the Abraham-Minkowski controversy.

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