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Magnetic solitons of spinor Bose-Einstein condensates in an optical lattice

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Magnetic solitons in spinor Bose-Einstein condensates confined in a one-dimensional optical lattice are studied by the Holstein-Primakoff transformation method. It is shown that due to the long-range light-induced and static magnetic dipole-dipole interactions, there exist different types of magnetic solitary excitations in different parameter regions. Compared to conventional solid-state materials, the parameters of this type of magnetic solitons in an optical lattice can be easily tuned by the above dipole-dipole interactions, which are highly controllable in experiments.

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I. INTRODUCTION

Recently, spinor Bose-Einstein condensates (BECs) trapped in optical potentials have received much attention in experimental [1] and theoretical fields [2]. Spinor Bose-Einstein condensates have internal degrees of freedom due to the hyperfine spin of the atoms. When spinor BECs are trapped in a magnetic potential, these degrees of freedom are frozen. However, when they are trapped in optical potential, the spin degrees of freedom are liberated and a rich variety of phenomena such as spin domains [3] and textures [4] have been observed. Recent studies show that spinor BECs, if localized in the optical lattices deep enough for the individual sites to be independent, can undergo a ferromagnetic-like phase transition that leads to a “macroscopic” magnetization of the condensates array [5,6]. Spinor BECs at each lattice site behave like spin magnets and can interact with each other through both the light-induced dipole-dipole interaction and the static magnetic dipole-dipole interaction. These site-to-site dipolar interactions can cause the ferromagnetic phase transition and the spin-wave excitation [5,7]. These phenomena are analogous to the ferromagnetism in solid-state physics, but occur with bosons instead of fermions. For fermions, the site-to-site interaction is caused mainly by the exchange interaction; the dipole-dipole interaction is small and can be neglected. For the spinor BEC in the optical lattice, the exchange interaction is absent and the individual spin magnets are coupled by the magnetic and light-induced dipole-dipole interaction [7]. Due to the large number of atoms, N, at each lattice site, these interactions are no longer negligible, despite the large distance, on the order of half of an optical wavelength, between sites. So, the spinor BECs in an optical lattice offer a totally new environment to study spin dynamics in periodic structures. In Ref. [7], the spin wave excitation of spinor BECs has been studied. However, the interaction between the spin waves has not been considered. This interaction would excite a magnetic soliton, which is also an important and interesting phenomenon in spin dynamics.

In this paper, we want to show that, due to the interactions between the spin waves of spinor BECs in a one-dimensional optical lattice, the magnetic solitons may exist, and this type of magnetic soliton can be easily tuned by the light-induced and the magnetic dipole-dipole interactions. The magnetic soliton, which describes localized magnetization, is an important excitation in the Heisenberg spin chain [8]. In the previous studies, the conventional magnetic solid materials containing many domains are often used, of which one possesses its own set of parameters such as magnetic anisotropy and barrier energy and is not easily adjusted. In addition, due to the difficultly of cooling the samples down to ultracold temperatures, thermal processes cannot be completely excluded. The defects and impurities of magnetic materials would also have important influences and need to be considered. But for the systems of spinor BECs in the optical lattice, the above difficulty is automatically excluded. So, these systems can give us a tool to investigate the nonlinear excitations in spin systems.

II. THE SPIN HAMILTONIAN FOR SPINOR BEC IN AN OPTICAL LATTICE

The dynamics of spinor BECs trapped in optical lattices are primarily governed by three types of two-body interactions: spin-change collision, magnetic dipole-dipole interaction, and light-induced dipole-dipole interaction. The Hamiltonian takes the following form:

$$H = \sum_n \int d\mathbf{r}\,\hat{\psi}_n^\dagger(\mathbf{r})\left[-\frac{\hbar^2\nabla^2}{2m} + V_L(\mathbf{r})\right]\hat{\psi}_n(\mathbf{r}) + \sum_{n,m,n',m'}\int d\mathbf{r}d\mathbf{r}'\,\hat{\psi}_n^\dagger(\mathbf{r})\hat{\psi}_m^\dagger(\mathbf{r}')[V^{\rm coll}_{nn'mm'}(\mathbf{r},\mathbf{r}') + V^{d-d}_{nn'mm'}(\mathbf{r},\mathbf{r}')]\hat{\psi}_{m'}(\mathbf{r}')\hat{\psi}_{n'}(\mathbf{r}) + H_B, \quad (1)$$

where $V_L(\mathbf{r})$ is the lattice potential, and the indices $n,m,n',m'=-F,\ldots,F$ denote the Zeeman sublevels of the ground state of the atoms with angular momentum $\mathbf{F}$. $V^{\rm coll}_{nn'mm'}(\mathbf{r},\mathbf{r}')$ describes the two-body ground-state collisions, $V^{d\text{-}d}_{nn'mm'}(\mathbf{r},\mathbf{r}')$ include magnetic dipole-dipole interaction and light-induced dipole-dipole interaction, and H_B represents the external magnetic interaction.

When the potential depth of the optical lattice is large enough, it is convenient to expand the spinor atomic field operator as $\hat{\psi}_m(\mathbf{r})=\Sigma_i\ \phi_i(\mathbf{r})C_m(i)$, where $\phi_i(\mathbf{r})$ is the condensate wave function for the ith microtrap and the operators $\hat{C}_m(i)$ satisfy the bosonic commutation relations $[\hat{C}_m(i),\hat{C}_n^\dagger(j)]=\delta_{mn}\delta_{ij}$. Under the tight-binding approximation and considering only the spin-dependent terms, we can construct the effective spin Hamiltonian from Eq. (1) [5,7],

$$H=\sum_i\left[\lambda_a'\hat{S}_i^2-\gamma_B\hat{S}_i\cdot\mathbf{B}-\sum_{j\neq i}J_{ij}(\hat{S}_i^-\hat{S}_j^+ +\hat{S}_j^+\hat{S}_i^-)-\sum_{j\neq i}J^{md}\hat{S}_i^z\hat{S}_j^z\right], \tag{2}$$

where $J_{ij}=J_{ij}^{ld}-\frac{1}{4}J_{ij}^{md}$, J_{ij}^{md} is a coefficient which represent magnetic dipole-dipole interaction, and J_{ij}^{ld} is a coefficient which represents light-induced dipole-dipole interaction. The collective spin operators are defined as $\hat{S}_i=\Sigma_{mn}\ \hat{C}_m^\dagger(i)\mathbf{F}_{mn}\hat{C}_n(i)$ with components $\hat{S}_i^{\{\pm,z\}}$, where $\mathbf{F}_{mn}$ is the matrix element of the angular moment $\mathbf{F}$. The direction of the magnetic field $\mathbf{B}$ is along the one-dimensional optical lattice, which we choose as the quantization axis $\mathbf{z}$, and $\mathbf{B}=B_z\mathbf{z}$. The parameter $\gamma_B=-\mu_B g_F$, with μ_B being the Bohr magneton and g_F the Landé g factor. The first term in Eq. (2) results from the spin-dependent interatomic collisions at a given site, with $\lambda_a'=(1/2)\lambda_a\int d^3r|\phi_i(\mathbf{r})|^4$. The last two terms describe the site-to-site spin coupling induced by the static magnetic and light-induced dipole-dipole interactions. The ground state of the Hamiltonian is $|\mathrm{g.s.}\rangle=|N,-N\rangle$ without interaction terms, where $N=\Sigma_i\ N_i$ is the total atomic number in the lattice. The total spin at site i has the expectation value $\langle\hat{S}_i^z\rangle=N_i\hbar$. Due to the large factor N_i, the magnetic dipole-dipole interaction in the optical lattice cannot be neglected. After the site-to-site coupling is considered, the transfer of the transverse spin excitation from site to site is allowed, resulting in the distortion of the ground-state spin structure. This distortion can propagate and hence generate magnetic soliton or spin wave along the atomic spin chain.

III. THE NONLINEAR EXCITATIONS OF SPINOR BEC IN AN OPTICAL LATTICE

For $J_{ij}\neq 0$, the transfer of transverse spin excitation from site to site is allowed, resulting in the distortion of the ground-state spin structure. This distortion can propagate and hence generate spin waves along the atomic spin chain. According to the Holstein-Primakoff (HP) transformation, we can introduce the local spin-deviation operator $\hat{n}=S-S^z$ with the eigenvalues $n=S-m$. So, increasing m decreases n and vice versa. For the state $|n\rangle$, $\hat{S}^-|n\rangle=\sqrt{(2S-n)(1+n)}|n+1\rangle$ and $\hat{S}^+|n\rangle=\sqrt{2S-(n-1)}\sqrt{n}|n-1\rangle$. We can define the creation and annihilation operators a and $a^\dagger$ on the state $|n\rangle$ as $a|n\rangle=\sqrt{n}|n-1\rangle$ and $a^\dagger|n\rangle=\sqrt{n+1}|n+1\rangle$ and the $a^\dagger a|n\rangle=n|n\rangle$. The operators $a^\dagger$ and a satisfy the following boson commutator relation: $[a,a^\dagger]=1,[a,a]=[a^\dagger,a^\dagger]=0$. According to the above relations, we can represent the spin operators by means of the Bose operators $a^\dagger$ and a according to HP transformations [9] $\hat{S}^+=(\sqrt{2S-a^\dagger a})a$, $\hat{S}^-=a^\dagger(\sqrt{2S-a^\dagger a})$, and $\hat{S}_z=(S-a^\dagger a)$. The square-root factors can be expanded in powers of $1/S$ as $\sqrt{2S-a^\dagger a}=\sqrt{2S}(1-a^\dagger a/4S+\cdots)$. If we take the first-order approximation, i.e., $\sqrt{2S-a_i^\dagger a_i}\approx\sqrt{2S}$ and substituting these transformations into Eq. (2), we can obtain the Hamiltonian which describes spin waves:

$$H=-\gamma_B NSB_z-J^zZNS^2+\sum_i\gamma_B B_z a_i^\dagger a_i+2ZJ^zS\sum_i a_i^\dagger a_i -2SJ\sum_{i\delta}(a_i^\dagger a_{i+\delta}+a_i a_{i+\delta}^\dagger). \tag{3}$$

The above Hamiltonian can be diagonalized by the Fourier transformation to $a_i^\dagger$, a_i and the dispersion relation of the spin wave can be obtained:

$$\hbar\omega_k=-\gamma_B B_z+\sum_{\delta>0}[4J^{md}S-8SJ\cos(\delta k)], \tag{4}$$

where we replace J_{ij}^{md},J_{ij}^{ld} with their average values J_{MD}, J_{LD}, and $J=J_{LD}-J_{MD}/4$. From Eq. (4), we can find that the dispersion relation of the spin wave derived by HP transformation is the same as the relation derived in Ref. [7]. In the optical lattice, the dispersion relation of the spin wave can be easily tuned by the light-induced and magnetic dipole-dipole interaction. But in the solid-state magnetic materials, the dispersion relation of the spin wave can be tuned with some difficulty by the exchange interaction, which is determined by properties of the magnetic material itself and is approximated as a constant.

The higher-order terms, i.e., $\sqrt{2S-a^\dagger a}\approx\sqrt{2S}(1-a^\dagger a/4S+\cdots)$, can introduce nonlinear interactions between spin waves, and then the Hamiltonian (2) becomes

$$\begin{aligned}H=&\lambda_a'NS(S+1)-\gamma_B B_zSN+\gamma_B B_z\sum_i a_i^\dagger a_i\\&-\sum_i\sum_{j\neq i}(J_{ij}^{md}S^2-J_{ij}^{md}Sa_j^\dagger a_j-J_{ij}^{md}Sa_i^\dagger a_i+J_{ij}^{md}a_i^\dagger a_i a_j^\dagger a_j)\\&-\sum_i\sum_{j\neq i}(2J_{ij}Sa_i^\dagger a_j+2J_{ij}Sa_ia_j^\dagger)+\frac{1}{2}\sum_i\sum_{j\neq i}J_{ij}\\&\times(a_i^\dagger a_i^\dagger a_i a_j+a_i^\dagger a_j^\dagger a_j a_j+a_j a_i^\dagger a_i^\dagger a_i+a_j^\dagger a_j a_j a_i^\dagger)+\cdots,\end{aligned} \tag{5}$$

where we omit the negligible fourth-order term.

The nonlinear interaction terms in Hamiltonian (5) generate the nonlinear magnetic excitations such as mixing of spin waves or magnetic solitons, depending on the initial setup of the state of the spin system. Here we are interested in the type of nonlinear excitations with magnetic solitons. To effectively observe the magnetic solitons in the nonlinear excitations, the ideal case is that the spin system in the optical lattice should initially be prepared in a spin-coherent state $|\psi\rangle=|\{\psi_l\}\rangle=\Pi_l\ |\psi_l\rangle$, with $|\psi_l\rangle=\exp(-|\psi_l|^2/2)\exp(-\psi_l a^\dagger)|0\rangle$. The vacuum state $|0\rangle$ is the ground state of the BEC in the optical lattice, i.e., $|0\rangle=|\mathrm{g.s.}\rangle=|N,-N\rangle$ [9]. Experimentally, such a spin-coherent state can be prepared by applying a rf pulse field along the transverse direction such as the y axis to the BEC in the ground state. If the rf pulse is short enough so that during the interaction of the spin system with the pulse, other interactions can be ignored, then in the rotating-wave frame with the Larmor frequency $\omega_L = \gamma_B B_z$, the rf pulse field would lead to a transformation

$$D(\psi) = \exp\left[i\theta(\hat{S}^+ - \hat{S}^-)/2\right] \approx \exp\left[\psi(a - a^\dagger)\right], \tag{6}$$

where θ is the area of the rf pulse and $\psi = i\theta\sqrt{2S}$. The transformation $D(\psi)$, operating at each lattice site, exactly leads to a coherent state $|\psi\rangle$ for the spin excitations.

Under the spin-coherent state and using the time-dependent variation principle, the nonlinear operator motion equation of Hamiltonian (5) can be transformed into the probability amplitude, Eq. (7), where $\psi_l = \langle\psi|a_l|\psi\rangle$ is the probability amplitude describing the nonlinear dynamics of coherent spin excitations on the lattice l,

$$\begin{aligned} i\hbar\frac{\partial \psi_l}{\partial t} = {} & (\gamma_B B_z + 4J_{MD}S - 8JS)\psi_l + 4JS(2\psi_l - \psi_{l+1} - \psi_{l-1}) \\ & - 2J_{MD}(|\psi_{l-1}|^2 + |\psi_{l+1}|^2)\psi_l \\ & + J(|\psi_{l+1}|^2\psi_{l+1} + |\psi_{l-1}|^2\psi_{l-1} + 2|\psi_l|^2\psi_{l+1} + 2|\psi_l|^2\psi_{l-1}) \\ & + J(\psi^+_{l+1} + \psi^+_{l-1})\psi_l^2, \end{aligned} \tag{7}$$

where we consider only the nearest-neighbor interactions (which is a good approximation for the BEC in a one-dimensional optical lattice as the large lattice constant and similar approximation has been adopted in Ref. [10]).

When the optical lattice is infinitely long and the spin excitations are in the long-wavelength limit, $\psi_l, \psi_{l+1}, \psi_{l-1} \to \psi(z,t)$ in the continuum limit approximation, we get the following soliton solutions in different regions [11] which can be described by the two parameters as $\alpha = 4JS = 4S(J_{LD} - J_{MD}/4)$ and $\beta = (4J_{MD} - 8J) = 6J_{MD} - 8J_{LD}$.

(1) When $J_{LD} < J_{MD}/4$, $\alpha < 0$, and $\beta > 0$, Eq. (7) has the following single dark soliton solution:

$$\psi(z,t) = \sqrt{-\frac{\beta}{\alpha}}\tanh\left[-\frac{\beta}{\sqrt{2}\alpha}(z - vt)\right]\exp\left[i(\gamma z - \omega_1 t)\right], \tag{8}$$

where $\hbar\omega_1 = \gamma_B B_z + 4J^z S - 8JS - \alpha v^2/4 - 2\beta^2/|\alpha|$ is the single dark soliton energy, and $\gamma = v/2$, where v is the velocity of the soliton. It is easy to see that the soliton energy $\hbar\omega_1$ is smaller than the spin-wave energy obtained above and that the amount is $2\beta^2/|\alpha|$ when $v \approx 0$. From Eq. (8), we can find that the amplitude and width of the magnetic soliton of spinor BEC in the optical lattice are $\Gamma = -\beta/\alpha = (3B_1 - 1)/[2S(B_1 - 1)]$ and $W = -\sqrt{2}\alpha/\beta = \sqrt{2}S(B_1 - 1)/[6(B_1 - 1/3)]$, respectively, where $B_1 = J_{MD}/4J_{LD} > 1$. So, the dynamic characteristics of the magnetic soliton existing in an optical lattice are more easily controlled than those in conventional solid magnetic materials. For example, when B_1 is increased (which can be realized simply by decreasing the light-induced dipole-dipole interaction J_{LD}), W is increased and Γ decreased. It should be noted that for a system of the optical lattice created by blue-detuned laser beams, the atoms are trapped in dark-field nodes of the lattice and the light-induced dipole-dipole interaction can be neglected, i.e., $J_{LD} = 0$. So the magnetic soliton will have invariable width and amplitude. This characteristic of the spin nonlinear excitation in the blue-detuned lattice might have potential applications in the future.

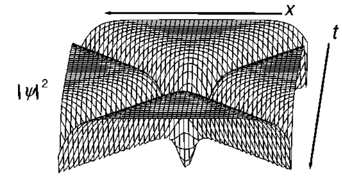

FIG. 1. The interaction of two dark magnetic solitons.

The magnetic soliton collision may be also be conveniently examined in the optical lattice. For example, the interaction of two dark magnetic solitons can be described by the following form:

$$|\psi(z,t)|^2 = \frac{2\rho^2}{\beta}\left|\frac{D_1}{N_1}\right|^2, \tag{9}$$

where

$$\begin{aligned} D_1 = {} & 1 + \exp(i2\eta_1 - 2\theta_1) + \exp(i2\eta_2 - 2\theta_2) \\ & + \exp\left[i2(\eta_1 + \eta_2) - 2(\theta_1 + \theta_2)\right]A, \end{aligned}$$

$$N_1 = 1 + \exp(-2\theta_1) + \exp(-2\theta_2) + \exp\left[-2(\theta_1 + \theta_2)\right]A,$$

$$\theta_1 = \mu_1\sqrt{-\alpha}(z - 2v_1\sqrt{-\alpha}t),$$

$$\theta_2 = \mu_2\sqrt{-\alpha}(z - 2v_2\sqrt{-\alpha}t),$$

$$\rho^2 = v_1^2 + \mu_1^2 = v_2^2 + \mu_2^2,$$

$$A = \frac{\rho^2 - v_1 v_2 - \mu_1\mu_2}{\rho^2 - v_1 v_2 + \mu_1\mu_2}, \tag{10}$$

ρ is background amplitude, $\exp(i2\eta_1) = (\lambda_1 + ik_1)^2/\rho^2$, and $\exp(i2\eta_2) = (\lambda_2 + ik_2)^2/\rho^2$. $v_1\sqrt{-\alpha}$, $v_2\sqrt{-\alpha}$ and $\mu_1\sqrt{-\alpha}$, $\mu_2\sqrt{-\alpha}$ refer to the velocity and width of magnetic solitons, respectively. For simplicity, here and in the following paragraph, we assume that the values of the initial phases and coordinates of the center of the two magnetic solitons are zero. The interactions of the two magnetic solitons are schematically shown in Fig. 1. By checking Fig. 1, we can see that the collision of two dark magnetic solitons in the optical lattice we considered is elastic and each of the magnetic solitons is restored to its initial form and velocity when it leaves the interaction region. In addition, the length of the interaction region of two dark magnetic solitons in the optical lattice is approximately proportional to $1/\sqrt{-\alpha}$ and can be tuned easily. For example, the interaction region would become small if we simply decrease the light-induced dipole-dipole interaction J_{LD}. It should be noted that in the above analysis, the collision of two magnetic solitons in the optical lattice is elastic. The inelastic collision would appear if the influences of higher-order terms in Eqs. (5) and (7) are considered.

(2) When $\frac{3}{4}J_{MD} > J_{LD} > \frac{1}{4}J_{MD}$, $\alpha > 0$, and $\beta > 0$, Eq. (7) has the following bright magnetic soliton solution:

$$\psi(z,t) = \sqrt{\frac{\beta}{\alpha}}\mathrm{sech}\left[\frac{\beta}{\alpha}(z - vt)\right]\exp\left[i(\gamma z - \omega_2 t)\right], \quad (11)$$

where $\hbar\omega_2 = \gamma_B B_z + 4J^z S - 8JS + \alpha v^2/4 - 2\beta^2/\alpha$, $\gamma = v/2$, v is the velocity of the soliton. Similarly, we can see that the energy of this bright magnetic soliton is also smaller than the spin wave and the amount is $2\beta^2/\alpha$ when $v \approx 0$. The amplitude and the width of the above bright magnetic soliton are $\Gamma = \beta/\alpha = (3B_2 - 1)/[2S(1 - B_2)]$ and $W = \alpha/\beta = 2S(1 - B_2)/(3B_2 - 1)$, respectively, where $1/3 < B_2 = J_{MD}/4J_{LD} < 1$. In this region, if we increase the light-induced dipole-dipole interaction, i.e., decrease B_2, Γ decreases and W increases. If we decrease the light-induced dipole-dipole interaction, i.e., increase B_2, Γ would increase and W would decrease.

The collision of the two bright magnetic solitons can be described by the solution as

$$|\psi(z,t)| = 2\sqrt{\frac{2}{\beta}}\left|\frac{N_2}{D_2}\right|, \quad (12)$$

where

$$D_2 = [(v_1 - v_2)^2 + \mu_1^2 + \mu_2^2] + B, \quad N_2 = C_1 + C_2,$$

$$B = 2\mu_1\mu_2 \tanh[\theta_1]\tanh[\theta_2] - 2\mu_1\mu_2\,\mathrm{sech}[\theta_1]\,\mathrm{sech}[\theta_2]\cos[\varphi_1 - \varphi_2],$$

$$C_1 = [(v_1 - v_2)^2 + \mu_1^2 - \mu_2^2]\mu_1\,\mathrm{sech}[\theta_1]\exp[-i\varphi_1] - 2i\mu_2(v_1 - v_2)\tanh[\theta_2]\mu_1\,\mathrm{sech}[\theta_1]\exp[-i\varphi_1],$$

$$C_2 = [(v_1 - v_2)^2 - \mu_1^2 + \mu_2^2]\mu_2\,\mathrm{sech}[\theta_2]\exp[-i\varphi_2] + 2i\mu_2(v_1 - v_2)\tanh[\theta_1]\mu_2\mathrm{sech}[\theta_2]\exp[-i\varphi_2],$$

$$\varphi_1 = \frac{2\mu_1}{\sqrt{\alpha}}z + 4(v_1^2 - \mu_1^2)t, \quad \varphi_2 = \frac{2\mu_1}{\sqrt{\alpha}}z + 4(v_2^2 - \mu_2^2)t,$$

$$\theta_1 = \mu_1\frac{4}{\sqrt{\alpha}}(z + 2v_1\sqrt{\alpha}t), \quad \theta_2 = \mu_2\frac{4}{\sqrt{\alpha}}(z + 2v_2\sqrt{\alpha}t),$$

$$\lambda_1 = v_1 + i\mu_1, \quad \lambda_2 = v_2 + i\mu_2, \quad (13)$$

where $2v_1\sqrt{\alpha}$, $2v_2\sqrt{\alpha}$, and $4\mu_1/\sqrt{\alpha}$, $4\mu_2/\sqrt{\alpha}$ are related to the velocity and width of magnetic solitons, respectively. The collision of two bright magnetic solitons in the optical lattice system we considered is elastic and the interaction region can also be tuned just as in the case of two dark magnetic solitons discussed above.

(3) When $J_{LD} > 4J_{MD}/3$, $\alpha > 0$, and $\beta < 0$, Eq. (5) has the solution

$$\psi(y,t) = 2\sqrt{\frac{\beta}{\alpha}}\mathrm{csch}\left[-\frac{\beta}{\alpha}(z - vt)\right]\exp\left[i(\gamma z - \omega_3 t)\right], \quad (14)$$

where $\hbar\omega_3 = \gamma_B B_z + 4J^z S - 8JS + \alpha v^2/4 - 2\beta^2/\alpha$, $\gamma = v/2$. But this solution is divergent and has no corresponding physical meaning. The magnetic soliton solution in this region may be caused by the contribution of the neglected higher terms and needs further study.

The magnetic solitons are localized excitations of spins which result from the nonlinear interactions between spin waves. In principle, the detection of the magnetic solitons can be carried out through the Raman technique by measuring the absorbtion spectrum of the Raman beams. This is similar to what was proposed to detect the spin waves in Ref. [7]. However, due to the localization of the magnetic soliton in space, it may be easier to detect magnetic solitons through the Raman absorption imaging of spin excitations at different times. One can use the same combination of a circularly polarized and a π-polarized Raman optical beam as that proposed in Ref. [7] and measure the absorption of the π-polarized Raman beam at different times. The absorption of the π-polarized Raman beam is proportional to the probability of the spin transition $|\langle \hat{S}^{+}\rangle|^2|\psi(z,t)|^2$. We see that one can image the space distribution of the spin excitations along the coherent spin chain of the condensed atoms in the optical lattice through the measurement of the Raman absorption. Comparing the images at different times, one can determine the properties of the magnetic solitons defined by Eq. (8) or (11).

IV. CONCLUSION

In conclusion, the nonlinear spin excitations of spinor BECs in an optical lattice have been studied using the HP transformation. The results show that the dark and bright magnetic solitons may exist in spinor BECs in optical lattices in different parameter regions. The width, the amplitude, and the length of the interaction region of these solitons can be adjusted by tuning the light-induced and magnetic dipole-dipole interactions. Being different from the conventional solid-state one-dimensional ferromagnetic chain, in which the magnetic solitons result from direct Coulomb interaction among electrons and the Pauli exclusion principle, i.e., exchange interactions. The magnetic solitons in spinor BECs in optical lattices are mainly caused by the magnetic and light-induced dipole-dipole interactions between different lattice sites, which cannot be neglected due to the significant Bose enhancement effect. Compared to the more conventional solid-state magnetic materials, these long-range interactions are highly controllable in the experiments, and the system of spinor BECs in optical lattice is an exceedingly clean system. This atomic system can provide us with a useful tool to study the fundamental static and dynamic aspects of magnetism and lattice dynamics.

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