Quantum dynamics of a vortex in a Josephson junction

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We investigate the tunneling character of vortex in an asymmetrical potential well with a finite barrier by using the periodic instanton method. We obtain the total decay rate which is valid for the entire range of temperature and show how it reduces to the appropriate results for the classical thermal activation at high temperatures, the thermally assisted tunneling at intermediate temperatures, and the pure quantum tunneling at low temperature. We can even give the exact definition of the "crossover" temperature and find experimental data to support our theoretical analysis.

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I. INTRODUCTION

The question of particlelike collective excitations, like domain walls, solitons, vortices, or fluxons, exhibiting quantum behavior has attracted much attention. Especially the escaping process of a particle from a potential well is a problem of great importance in almost all areas of physics.[1–7]

Vortices occur naturally in a wide range of gases and fluids, from macroscopic to microscopic scales.[8–10] Two kinds of vortices can develop in superconductors. The first, the Abrikosov vortex, penetrates certain (type II) superconductors above a critical value of applied magnetic field. It can hop between pinning sites under thermal activation, causing dissipation in current-carrying wires and generating noise in sensors such as superconducting quantum interference devices (SQUIDs). Quantum tunneling of Abrikosov vortices remains controversial. The second vortex, the Josephson vortex, exists in a Josephson junction, formed by a sandwich of two superconducting layers and a thin insulating layer, through which electrons in the form of Cooper pairs can tunnel coherently. In a current-biased Josephson junction, swirling currents generate vortices of flux. Vortex motion in current-biased Josephson junction systems has been the subject of much theoretical and experimental work.[11–14]

Recently, Wallraff *et al.*[8] showed that a single pinned vortex in a current-biased annular junction subject to an in-plane field can undergo macroscopic quantum tunneling to escape from a potential well; they also showed that the vortex's energy in the controllable well is quantized. In their experiment, they made an annular junction between two narrow rings of the superconductor niobium, stacked one on top of the other. The flux in each ring is quantized in units of $h/2e$ (where h is Plank's constant and e is the charge on an electron); when this flux quantum number differs between the two rings by one unit, the difference is manifested as a single vortex in the junction. If a magnetic field is applied in the plane of the annulus, at an angle to the magnetic moment of the vortex, the potential energy of the system is proportional to the cosine of the angle. If an external current is then applied to the junction, across the two superconducting layers, the resulting imbalance in the tunneling currents produces a force on the vortex to escape and move along the barrier.

In this paper we investigate the quantum tunneling of a vortex in a long Josephson junction by using the periodic instanton method which is well known as a powerful tool for dealing with quantum tunneling phenomena.[15,16] In our model, we simplify a single vortex behavior in an annular junction subject to an in-plane field as a particle in a tilted washboard potential. We first calculate the total decay rate for all temperatures and show how it reduces to the appropriate results for the classical thermal activation at high temperatures, the thermally assisted tunneling at intermediate temperatures, and the pure quantum tunneling at low temperatures.

The rest of the paper is as follows: In Sec. II we gain the base mode of our path-integral approach from the master equation. In Sec. III we recall from Ref. 16 the bounce with nonzero energy and again give the main process of the calculation. Results and comparison with experiments are discussed in Sec. IV. The analysis of quantum-classical transition property is in Sec. V.

II. THE MASTER EQUATION IN CURRENT-BIASED JOSEPHSON JUNCTION

The dynamics of a current-biased Josephson tunnel junction can be described by the master equation.[17,18] The behavior of the superconducting tunnel junction is described by the Josephson relations

$$I_J = I_c \sin \phi, \quad V = \left(\frac{\Phi_0}{2\pi}\right)\frac{d\phi}{dt}. \tag{1}$$

Here I_J is the tunneling current of Cooper pairs flowing through the junction, I_c the critical current, V the voltage across the junction, ϕ the phase difference of the superconducting order parameter, and $\Phi_0 = h/2e \approx 2.07\times 10^{15}$ Wb the flux quantum. The classical equation describing the system is

$$I = C\frac{dV}{dt} + \frac{V}{R} + I_J, \qquad (2)$$

where I is the bias current, C the capacitance, and R the resistance characterizing the dissipation. Inserting the Josephson relations into the classical equation one obtains

$$C\left(\frac{\Phi_0}{2\pi}\right)^2\frac{d^2\phi}{dt^2} + \frac{1}{R}\left(\frac{\Phi_0}{2\pi}\right)^2\frac{d\phi}{dt} = -\frac{\partial}{\partial\phi}\left[-I_c\left(\frac{\Phi_0}{2\pi}\right)\left(\cos\phi + \frac{I}{I_c}\phi\right)\right]. \qquad (3)$$

This equation is identical with the equation of motion of a particle with mass $M=(\Phi_0/2\pi)^2C$, subject to damping and a fluctuating force, and move in the one-dimensional potential $\widetilde{U}(\phi)$, commonly known as the washboard potential: $\widetilde{U}(\phi) = -I_C(\Phi_0/2\pi)(\cos\phi + I/I_c\phi)$.

The component of the potential periodic in the vortex coordinate ϕ is due to the interaction of the vortex magnetic moment with the external magnetic fields. The tilt of the potential is proportional to the Lorentz force acting on the vortex which is induced by the bias current applied to the junction. The rate at which the particle escapes from the potential depends on details of the shape of the potential.

Experimentally, the escape rate of a vortex from the zero-voltage state to the finite-voltage state was measured. In the mechanical analog this corresponds to measuring the escape of the particle from one of the washboard potential wells. The escape might occur either by thermal fluctuations or by quantum tunneling. Since in the weak-damping limit the particle is unlikely to be retrapped in another well, we are interested in only one potential well of the washboard potential. We can approximate it very well, for bias currents close to the critical current, by a cubic parabola,

$$U(\phi) = 3U_0\left(\frac{\phi}{\phi_0}\right)^2\left(1 - \frac{2\phi}{3\phi_0}\right). \qquad (4)$$

The height U_0 and the position of the maximum ϕ_0 are given in terms of the parameters of the junction by $U_0=2I_c[\phi_0/2\pi][(1-I'^2)^{1/2}-I' \arccos I']$, $\phi_0 = 2\sqrt{3}[1-I' \arccos I'/(1-I'^2)^{1/2}]$, where $I'=I/I_c$. Increasing the bias current I through the junction corresponds to reducing the height U_0 of the barrier.

The existence of tunneling results in a complex vortex energy E (see Fig. 1). The decay rate Γ of the vortex states is defined as the imaginary part of the complex energy, $\Gamma = 2/\hbar\,\mathrm{Im}\,E$. If $|\Psi\rangle$ denotes an eigenstate of the Hamiltonian H with energy E, the transition amplitude A from the state $|\Psi\rangle$ to itself—the "survival probability" of $|\Psi\rangle$—in the presence of quantum tunneling over Euclidean time 2β reads

$$A = \langle\Psi|e^{-2H\beta}|\Psi\rangle = e^{-2E\beta}. \qquad (5)$$

The amplitude A can also be calculated with the help of the path-integral method. Comparing the defined transition amplitude A from Eq. (5) with the path-integral result in the next section, we can find the decay rate Γ.

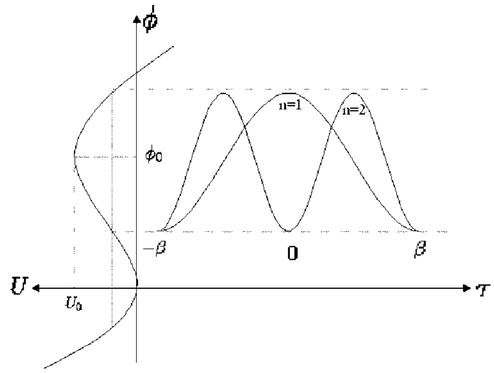

FIG. 1. A particle with energy E moving in the cubic potential given by Eq. (4); trajectories of the nonvacuum instantons (the case $n=1$ and $n=2$) given by Eq. (9).

III. TUNNELING OF VORTEX

The Lagrangian for a 0+1 dimension scalar field $\phi(t)$ is $L=\frac{1}{2}M(d\phi/dt)^2-U(\phi)$. By using Euclidean time $\tau=it$ we can rewrite L as

$$L_E[\phi(\tau),\dot{\phi}(\tau)] = -L = \frac{1}{2}M\left(\frac{d\phi}{d\tau}\right)^2 + U(\phi), \qquad (6)$$

and the classical action is

$$S_E(\phi) = \int_{\tau_i}^{\tau_f} L_E[\phi(\tau),\dot{\phi}(\tau)]d\tau. \qquad (7)$$

The classical solution ϕ_c which minimizes the action with Euclidean time τ satisfies the equation

$$\frac{1}{2}M\left(\frac{d\phi_c}{d\tau}\right)^2 - U[\phi_c(\tau)] = -E. \qquad (8)$$

The periodic instanton represents the pseudoparticle configuration responsible for tunneling under the barrier at energy E. Tunneling out of a vortex state in a potential $U(\phi)$ can be treated as motion with imaginary time $\tau=it$ in the corresponding inverted potential. The corresponding "periodic instanton" solution[19]

$$\phi_c(\tau) = \phi_1 - (\phi_1 - \phi_2)\mathrm{sn}^2(u|k) \qquad (9)$$

is periodic with period $\mathcal{T}$, $\gamma(k')\mathcal{T}=n2K(k)$, $n=1,2,3,\ldots$, and $\phi_c(\tau+\mathcal{T})=\phi_c(\tau)$, where $\phi_1(E)>\phi_2(E)>\phi_3(E)$ denote three roots of the equation $U(\phi)=E$, ϕ_1 and ϕ_2 denote the turning points of the instanton motion in inverted potential. $\mathrm{sn}(u|k)$ denotes a Jacobian elliptic function with the modulus $k = \sqrt{(\phi_1-\phi_2)/(\phi_1-\phi_3)}$, where $u=\gamma(k')\tau$, $\gamma(k') = \sqrt{U_0(\phi_1-\phi_3)/M\phi_0^3}$, $k'=\sqrt{1-k^2}$ is the complementary modulus of k.

Considering the small fluctuation about the classical solution $\phi_c(\tau)$ is

$$\phi(\tau) = \phi_c(\tau) + \eta(\tau), \qquad (10)$$

and correspondingly

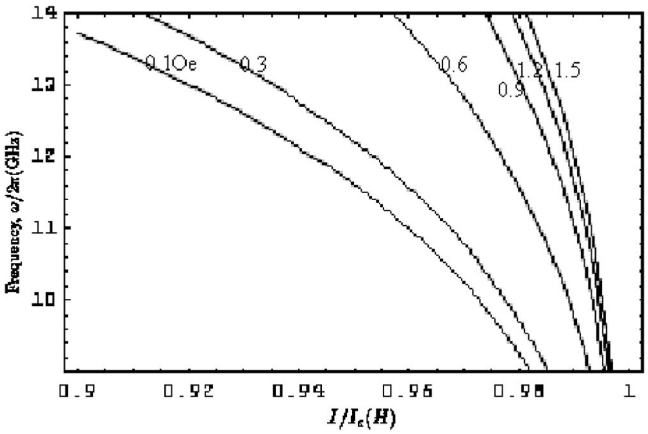

FIG. 2. Microwave frequency $\omega/2\pi$ vs normalized bias current $I/I_c(H)$ at $T=25$ mK with Wallraff experimental parameters.

$$S_E(\phi) = S_E[\phi_c(\tau) + \eta(\tau)] \simeq S_c(\phi) + \delta S_E, \qquad (11)$$

where $S_c(\phi)=\int_{\tau_i}^{\tau_f}\{\frac{1}{2}M\dot{\phi}_c^2(\tau)+U[\phi_c(\tau)]\}d\tau$, and $\delta S_E = \frac{1}{2}\int_{\tau_i}^{\tau_f}\eta\hat{M}\eta d\tau$ with $\hat{M}=-d^2/d\tau^2+U''(\phi_c)$.

The amplitude A can also be calculated with the help of the path integral method. We rewrite it

$$A = \int \psi_E^{\;*}(\phi_f)\psi_E(\phi_i)K(\phi_f,\tau_f;\phi_i,\tau_i)d\phi_f d\phi_i, \qquad (12)$$

with $\phi_f=\phi(\tau_f)$, $\phi_i=\phi(\tau_i)$, and $\tau_f-\tau_i=2\beta$. Thus ϕ_i and ϕ_f denote the end points of the instanton motion which tend to the turning point ϕ_2. The Feynman propagator from ϕ_i to ϕ_f resulting from the instanton motion is defined by

$$K(\phi_f,\tau_f;\phi_i,\tau_i) = \sum_{n=0,1,2\ldots} K_n(\phi_f,\tau_f;\phi_i,\tau_i),$$

$$K_n(\phi_f,\tau_f;\phi_i,\tau_i) = \int_{\phi_i\to\phi_f} D\{\phi\}\exp[-S_E(\phi)]. \qquad (13)$$

The zero bounce contribution. $n=0$ means there is no tunnel occurring which equals the harmonic oscillator in infinite barrier, i.e., $\frac{1}{2}M\omega_0^2\phi_0^2=3U_0$ and the zero bounce contributes to the real part of the complex energy

$$A_0 = e^{-2E_{cl}\beta}, \qquad (14)$$

where $E_{cl}=(m+\frac{1}{2})\hbar\omega_0$, $m=o,1,\ldots$ and $\omega_0=\sqrt{6U_0/M\phi_0^{\;2}}$. Both bias current and applied magnetic field scale the energy level separation $\Delta E=\hbar\omega_0\propto[1-I/I_c(H)^2]^{1/4}$. We draw the microwave frequency versus normalized bias current in Fig. 2 with Wallraff experiment parameters. In fact, the results of Fig. 2 just give the theoretical dashed lines in Fig. 3(b) of Ref. 8. At low temperature and in the absence of microwave radiation, the occupation of the excited state is very small. By irradiating the sample with microwaves, the vortex can be excited resonantly from the ground state to the first excited state. Then obviously a lower bias current corresponding to the tunneling out to the first excited state will be detected [see Fig. 3(a) in Ref 8].

The one bounce contribution. Setting $\mathcal{T}=2\beta$ and taking $n=1$, we have $\gamma(k')\beta=K(k)$, where β is half the period of the motion of the pseudoparticle as indicated in Fig. 1. As the energy tends to zero with $k\to 1$, the periodic instanton solution reduces the usual vacuum bounce.[19] On the other hand, as the energy approaches the top of the barrier $E\to U_0$ with $k\to 0$, the solution becomes the trivial configuration $\phi_c=\phi_0$. This trivial solution is called a sphaleron. The nonvacuum bounce thus interpolates between the vacuum bounce and this sphaleron.

The necessary boundary conditions for $\eta(\tau)$, where $\eta(\tau)$ denotes the small deviation of ϕ from the classical trajectory with end points held fixed, are $\eta(\tau_i)=\eta(\tau_f)=0$. From the expressions of K_n in Eq. (13) and S_E in Eq. (11) we obtain $K_1=e^{-S_c(\phi)}I_\eta$, where I_η is the functional integral $I_\eta=\int_{\eta(\tau_i)=0}^{\eta(\tau_f)=0}D\{\eta\}e^{-\delta S_E}$. We can evaluate the expression with the help of tables of integrals in Ref. 20 and find with $\tau_i=-\beta$, $\tau_f=\beta$, and $\beta=K(k)/\gamma(k')$, $S_c(\phi)=W[\phi(\tau_f),\phi(\tau_i),E_{cl}]+2E_{cl}\beta$, where $W=\frac{8}{15}\sqrt{MU_0/\phi_0^{\;3}}(\phi_1-\phi_3)^{5/2}[(1-k^2)(k^2-2)K(k)+2(k^4-k^2+1)E(k)]$, where $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kinds.

The Feynman propagator of the path integral is divergent because the velocity of instanton $N(\tau)=d\phi_c/d\tau=-2\sqrt{U_0/M}(\phi_1-\phi_2)\sqrt{\phi_1-\phi_3}sn(u|k)cn(u|k)dn(u|k)$ vanishes at the turning points, i.e., $cn[\pm K(k)|k]=o$. The transition amplitude has to be finite and this singularity must be smoothed out by turning point integrations of $d\phi_i$ and $d\phi_f$. We use the following relations established in Ref. 16: $I_\eta=-i[1/2\pi]^{1/2}[N(\phi_f)/N(\phi_i)][\partial^2S_c(\phi_f,\phi_i;\beta)/\partial\phi_f^2]^{1/2}$.

Further, we replace the wave functions $\psi_E(\phi_i)$ and $\psi_E(\phi_f)$ in Eq. (12) by their leading WKB approximations, and expand the action in powers of $\phi_2-\phi_c(\beta)$ up to the second order, which corresponds to the one loop approximation. Thus we write $S_c(\phi_f,\phi_i;\beta)=S_c[\phi(\beta),\phi(-\beta);\beta]+\frac{1}{2}\partial^2S/\partial\phi(\beta)^2[\phi_f-\phi(\beta)]^2+\cdots$. The WKB approximations of the wave functions are given by $\psi_E(\phi_f)=[C\exp(-\int_{\phi_2}^{\phi_f}\dot{\phi}d\phi)]/\sqrt{\dot{\phi}_f}$, $\psi_E(\phi_i)=[C\exp(-\int_{\phi_2}^{\phi_i}\dot{\phi}d\phi)]/\sqrt{\dot{\phi}_i}$. The normalization constant C is defined by $C=(2\int_{\phi_3}^{\phi_2}d\phi/\sqrt{2/M[E-U(\phi)]})^{-1/2}$, where the integration extends from turning point to turning point across the nontunneling domain (i.e., the region of the harmonic-oscillator approximation). Evaluating C one obtains $C=[\sqrt{U_0/M\phi_0^3}\sqrt{\phi_1-\phi_3}/2K(k')]^{1/2}$.

Using one loop expansion of the action and completing the turning point integration, we can obtain one instanton contribution

$$A_1 = -2i\beta e^{-W-2E\beta}\sqrt{\frac{U_0}{M\phi_0^{\;3}}}\frac{\sqrt{\phi_1-\phi_3}}{2K(k')}. \qquad (15)$$

The one instanton contribution is that of the classical configuration with period $\mathcal{T}=n2K(k)/\gamma(k')$ with $n=1$. For $n=2$ there are two instantons moving from $-\beta$ to β with "positions" $\tau_0=\pm K(k)/\gamma(k')$ (see Fig. 1). The contribution A_n to the transition amplitude arising from n instantons can be calculated analogously

$$A_n = (-i)^n \frac{(2\beta)^n}{n!} e^{-nW-2E\beta}\left(\sqrt{\frac{U_0}{M\phi_0^{\,3}}}\frac{\sqrt{\phi_1-\phi_3}}{2K(k')}\right)^n. \quad (16)$$

The total transition amplitude A is obtained by summation,

$$A = \sum_n A_n = e^{-2E\beta}\exp\left(-i2\beta\sqrt{\frac{U_0}{M\phi_0^{\,3}}}\frac{\sqrt{\phi_1-\phi_3}}{2K(k')}e^{-W}\right). \quad (17)$$

We can get the imaginary part of energy E by comparing the above amplitude A in Eq. (17) with defined amplitude A in Eq. (5). The decay rate of vortex state with energy E can be written as

$$\Gamma = \frac{\omega_E}{2\hbar K(k')}e^{-W}, \quad (18)$$

where $\omega_E = \sqrt{2(\phi_1-\phi_3)/3\phi_0}\omega_0$ is the energy dependent frequency. We emphasize that this compact formula is valid for the entire region of energy $0 < E < U_0$. It can be applied to any excited states from bottom to top of the well.

In low energy limit $E \ll U_0$, introducing a harmonic approximation $E_m = m\hbar\omega_0 + E_0$, we can evaluate the decay rate Γ_m of mth excited state,

$$\Gamma_m = \frac{1}{m!}\left(\frac{432U_0}{\hbar\omega_0}\right)^m \Gamma_0. \quad (19)$$

When $m=0$, Γ_m reduces to the decay rate Γ_0 of the vortex state with energy E_0 and $\Gamma_0 = 12\omega_0\sqrt{6U_0/\pi\hbar\omega_0}e^{-36U_0/5\hbar\omega_0}$.

We can find that the decay rate depends on the well depth. In a future experiment, we can increase the bias current through the junction corresponding to reducing the height of the barrier. The tunneling rate could be increased by lowering the well depths.

IV. TEMPERATURE DEPENDENCE

Taking a statistical average of the decay rates of different vortex states, $\Gamma(T) = 1/Z_0\Sigma_m\Gamma_m e^{-E_m/k_BT}$, where $Z_0 = \Sigma_m e^{-E_m/k_BT}$, we can obtain the decay rate of temperature dependence

$$\Gamma(T) = \Gamma_0(1-e^{-\hbar\omega_0/k_BT})e^{432U_0/\hbar\omega_0 e^{-\hbar\omega_0/k_BT}}, \quad (20)$$

where k_B is the Boltzmann constant. With experimental parameter $\omega_0 = 2\pi\nu_{01} = 75$ GHz for ν_{01} between 10 and 13 GHz given in Ref. 8 and $U_0 = 2.5\hbar\omega_0$ for two energy levels in the well, Fig. 3 shows three distinct regions with different behavior for $\Gamma(T)$. At high temperature, the decay is thermally activated and exhibits the expected Arrhenius law,[21] $\Gamma_{AR} = \omega_0/2\pi e^{-U_0/k_BT}$. At intermediate temperature, we observe "thermally assisted " tunneling, in which atoms are thermally activated to excited quantum state in the well, and then decay out from the well by quantum tunneling. At lower temperature, pure quantum tunneling from the trapped ground state is dominate, which is controlled by the "vacuum" instanton trajectory. Hence the ordinarily definition of the "crossover" temperature is $T_{cr} = \hbar\omega_0/2\pi k_B$, below which quantum tunneling dominates. The decay rate is independent on temperature

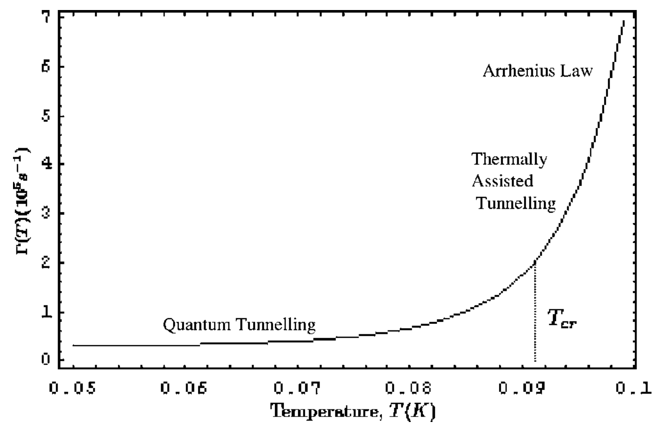

FIG. 3. The temperature dependence of the decay rate $\Gamma(T)$ with experimental parameters $\omega_0 = 75$ GHz and $U_0 = 2.5\hbar\omega_0$.

if $T < T_{cr}$, whereas it will increase appropriately with temperature if $T > T_{cr}$. We find $T_{cr} = 92$ mK which is consistent with the "crossover" temperature extracted from the temperature dependence of the switching current distributions in Ref. 8. In the following section, we will give another definition of the T_{cr} using our method, which will help in getting a deeper understanding of the conception.

In order to maker clearer the meaning of $\Gamma(T)$, we repeat the process of the experiment.[8] Wallraff *et al.* investigate the escape process by ramping up the current that is flowing through the annular junction until they see a jump in the voltage across the junction, which corresponds to the vortex beginning to rotate rapidly around the annulus (the voltage is proportional to the vortex velocity). This depinning of the vortex and its subsequent escape from the well is a stochastic process, so repeated measurements yield a distribution of switching currents at which depinning occurs. As the temperature is lowered, the width of this distribution shrinks, indicating a crossover from the classical region to macroscopic quantum tunneling in the quantum regime. We assume that we have an initial thermal distribution of the vortices (the decay principle will be consistent with the statistical experimental results); the population should decay as

$$N(t) = \frac{N_0}{Z_0}\sum_m e^{-\Gamma_m t - E_m/k_BT}, \quad (21)$$

where N_0 is the initial population of trapped vortices. Since $N(t)$ is a sum of exponential decay factors, it does not decay exponentially itself, i.e., $N(t)$ is not a straight line if we plot $\log N(t)$ as a function of time. There is not a single slope, and therefore it cannot be assigned a single decay rate. At long times, the curve approaches a straight line, with the corresponding rate given by Γ_0, i.e., the slowest channel dominates the long time behavior. In fact, the initial slope at time $t=0$ corresponds to a rate of $\Gamma(T)$ as calculated, meaning that $\Gamma(T)$ is simply the initial rate of decay of the population. The above discussion is valid if there is no thermalization of the population during decay, which is a situation that the experimentalist can realize. On the other hand, it is also possible to intentionally introduce intrawell transitions between the dif-

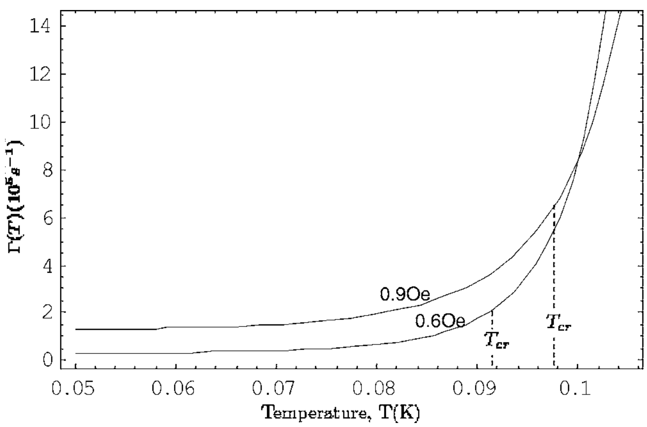

FIG. 4. The decay rate $\Gamma(T)$ vs T for two values of field. The crossover temperatures are indicated.

ferent levels, such that thermalization can be established in a time t_c. In this case, if t_c is short compared to $1/\Gamma(T)$, the population will decay at $\Gamma(T)$ at all times. If it is larger than $1/\Gamma(T)$, the population will decay initially nonexponentially until time t_c, after which the population will decay at $\Gamma(T')$, where T' is some temperature below T. There is a cool down of the population due to the initial decay.

To probe the $\Gamma(T)$ versus the temperature for a different external condition we draw Fig. 4. In order to have a qualitative analysis about Fig. 2(b) in Ref. 8, we use the experiment parameters $I_c(H)$. From our Fig. 4, we can find a value of T_{cr} about $90 \pm mk$, which is consistent with the saturation temperature in Fig. 2 of Ref. 4. In Fig. 4, we can see in a higher field the quantum region corresponds to a higher decay rate. That indicates the quantum tunneling is also dominated by the potential. If a higher field is applied to the junction, which corresponds to the vortex having a higher harmonic oscillator frequency and less shallow effective potential, the quantum tunneling will get a higher decay rate. This is the reason for Fig. 2(b) in Ref. 8 having two branches in the quantum region.

V. QUANTUM-CLASSICAL TRANSITION

The second derivative of the action, which is proportional to dE/dT, can be interpreted as the specific heat of the system.[22] Our model, allowing explicit calculation of periodic instantons and the corresponding evaluation of the action, can be studied with regard to phase transitions from classical to quantum behavior.[22,23] When the vortex escape from the potential under certain in-plane field, the second-order transition from thermal activation to macroscopic quantum tunneling should be observed as the temperature is lowered. The thermodynamic action, i.e., that of the sphaleron at the top of the barrier, and the action are correspondingly given by $S_0 = (U_0\hbar)/(k_B T)$ and $S_T = E/T + W$. Figure 5 displays the behavior of S_T and S_0 versus temperature T. We emphasize that the temperature of the point of intersection is the "cross" temperature T_{cr}. In Fig. 5(b), arrows show the actual dependence of $S(T)$ as the temperature is lowered and one can clearly see the typically smooth behavior of a second order transition from the thermal to the quantum regime as the temperature is lowered.

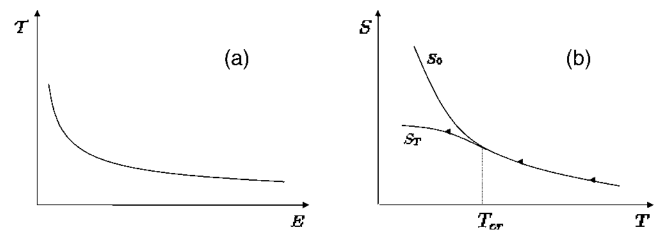

FIG. 5. (a) Monotonic dependence of $\mathcal{T}$ on E. (b) Arrows show the actual dependence of $S(T)$ as temperature is lowered.

In conclusion, we have calculated the total decay rate over the entire range of temperature of the vortex, which is confined in a current-biased annular junction subject to an in-plane field. We also pointed out that the system acquires a second order transition.

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[1] A. Shnirman, E. Ben-Jacob, and B. Malomed, Phys. Rev. B **56**, 14677 (1997).
[2] W. M. Liu, W. B. Fan, W. M. Zheng, J. Q. Liang, and S. T. Chui, Phys. Rev. Lett. **88**, 170408 (2002).
[3] G. Kälbermann, Phys. Rev. E **55**, R6360 (1997).
[4] J. A. González, A. Bellorín, and L. E. Guerrero, Phys. Rev. E **60**, R37 (1999).
[5] A. V. Ustinov, C. Coqui, A. Kemp, Y. Zolotaryuk, and M. Salerno, Phys. Rev. Lett. **93**, 087001 (2004).
[6] N. Grønbech-Jensen, M. G. Castellano, F. Chiarello, M. Cirillo, C. Cosmelli, L. V. Filippenko, R. Russo, and G. Torrioli, Phys. Rev. Lett. **93**, 107002 (2004).
[7] M. H. Devoret, J. M. Martinis, and J. Clarke, Phys. Rev. Lett. **55**, 1908 (1985).
[8] A. Wallraff *et al.*, Nature (London) **425**, 155 (2003).
[9] M. V. Fistul, A. Wallraff, Y. Koval, A. Lukashenko, B. A. Malomed, and A. V. Ustinov, Phys. Rev. Lett. **91**, 257004 (2003).
[10] J. Clarke, Nature (London) **425**, 133 (2003).
[11] A. van Oudenaarden, S. J. K. Vardy, and J. E. Mooij, Phys. Rev. Lett. **77**, 4257 (1996).
[12] T. S. Tighe, A. T. Johnson, and M. Tinkham, Phys. Rev. B **44**, 10286 (1991).
[13] M. V. Fistul and A. V. Ustinov, Phys. Rev. B **68**, 132509 (2003).
[14] Z. A. Xu *et al.*, Nature (London) **406**, 486 (2000).
[15] Y. B. Zhang, Ph.D. thesis, Institute of Physics, Chinese Academy of Sciences, 1998.

[16] J. Q. Liang and H. J. W. Müller-Kirsten, Phys. Rev. D **50**, 6519 (1994).
[17] P. Kopietz and S. Chakravarty, Phys. Rev. B **38**, 97 (1988).
[18] A. Wallraff *et al.*, Rev. Sci. Instrum. **74**, 3740 (2003).
[19] W. Zwerger, Z. Phys. B: Condens. Matter **51**, 301 (1983).
[20] P. F. Byrd and M. D. Friedman, *Handbook of Elliptic Integarls for Engineers and Scientists*, 2nd ed. (Springer, New York, 1971).
[21] U. Weiss, *Quantum Dissipative Systems* (World Scientific, Singapore, 1993).
[22] E. M. Chudnovsky, Phys. Rev. A **46**, 8011 (1992).
[23] J. Q. Liang, H. J. W. Muller-Kirsten, D. K. Park, and F. Zimmerschied, Phys. Rev. Lett. **81**, 216 (1998).