Nonlinear magnetization dynamics in a ferromagnetic nanowire with spin current

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We investigate the effect of spin transport on nonlinear excitations in a ferromagnetic nanowire with nonuniform magnetizations. In the presence of spin-polarized current, the exact soliton solutions propagating along the wire's axis are obtained in two cases of high or low quality factors (Q). The current can change the velocities of magnetic solitons and affect the solitons' collision with phase shift. Also, ac current induces vibration of the center of the solitons.

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I. INTRODUCTION

In magnetic multilayers, the spin-polarized current causes many unique phenomena, such as spin-wave excitation,[1] magnetization switching and reversal,[2–6] and enhanced Gilbert damping.[7] These phenomena attribute to the spin-transfer effect, proposed by Slonczewski[8] and Berger.[9] Macroscopically, the magnetization dynamics in the presence of spin-polarized current can be described by a generalized Landau-Lifshitz-Gilbert (LLG) equation, including terms related to spin-polarized current, spin accumulation, and spin-transfer torque. Bazaliy *et al.* considered spin-polarized current in half-metal and reduced the current effects as a topological term in the LLG equation.[10] The spin torque has different forms for uniform magnetization and nonuniform ones. In the uniform case, such as spin-valve structure, the spin torque is $\boldsymbol{\tau}_a=-(a_J/M_s)\mathbf{M}\times(\mathbf{M}\times\hat{\mathbf{M}}_p)$, where $\mathbf{M}$ is the magnetization of the free layer, $\hat{\mathbf{M}}_p$ is the unit vector along the direction of the magnetization of the pinned layer, M_s is the saturation magnetization, and a_J is a model-dependent parameter and is proportional to the current density.[8] For the nonuniform magnetization, Li and Zhang gave a spin torque taking form $\boldsymbol{\tau}_b=b_J\partial\mathbf{M}/\partial x$, where b_J depends on the materials parameters and current.[11] The spin torque is very different from precession and damping terms in the LLG equation. It can induce not only precession but also damping. Due to the unique property of the spin torque, the magnetization dynamics has been extensively studied in spin-valve pillar structures,[12,13] magnetic nanowires geometry,[14] and point-contact geometry.[15]

Nonlinear excitations are general phenomena in magnetic ordered materials. Extensive investigations have been made in insulated thin films for easy excitation and detection, such as YIG.[16] In confined magnetic metals, nonlinear excitations are also interesting due to the interaction between spin-polarized current and local magnetizations. The solitary excitations of isotropic ferromagnets in the present of spin-polarized current is well studied using the inverse scattering method.[17]

In this paper, we will discuss the linear and nonlinear magnetization dynamics in a uniaxial ferromagnetic nanowire injected with current. By stereographic projection of the unit sphere of magnetization onto a complex plane,[18] we transform the LLG equation into a nonlinear equation of complex function and obtain two kinds of soliton solutions propagating along the wire's axis for a high-Q model and a low-Q one $[Q=H_k/(4\pi M_s)$ is the quality factor$]$. From these solutions, we study the effect of a spin-polarized current on the nonlinear excitation of the magnetization in the ferromagnetic metal nanowire.

II. HIGH-Q CASE

We consider a very long ferromagnetic nanowire with a uniform cross section, as shown in Fig. 1. To excellent approximation, this nanowire can be viewed as infinite in length. The electronic current flows along the long length of the wire, defined as the x direction. The z axis is taken as the directions of the uniaxial anisotropy field and the external field. Also, we assume the magnetization is nonuniform only in the direction of current. The current injected in the nanowire can be polarized and produces a spin torque acting on the local magnetization, which is written as $\boldsymbol{\tau}_b=b_J\partial\mathbf{M}/\partial x$, where $b_J=Pj_e\mu_B/(eM_s)$, P is the spin polarization of the current, j_e is the electric current density and flows along the x direction, μ_B is the Bohr magneton, and e is the magnitude of electron charge.[11] So, the modified LLG equation with this spin torque is

$$\frac{\partial\mathbf{M}}{\partial t}=-\gamma\mathbf{M}\times\mathbf{H}_{\text{eff}}+\frac{\alpha}{M_s}\mathbf{M}\times\frac{\partial\mathbf{M}}{\partial t}+\boldsymbol{\tau}_b, \tag{1}$$

where γ is the gyromagnetic ratio, M_s is the saturation magnetization, α is the Gilbert damping parameter, and $\mathbf{H}_{\text{eff}}$ is the effective magnetic field including the external field, the

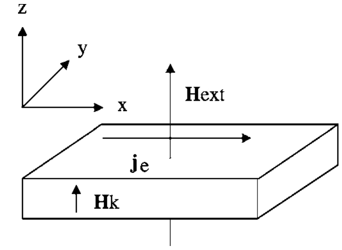

FIG. 1. Geometry of the nanowire.

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anisotropy field, the demagnetization field, and the exchange field. This effective field can be written as $\mathbf{H}_{\text{eff}} = (2A/M_s^2)\partial^2\mathbf{M}/\partial x^2 + [(H_K/M_s - 4\pi)M_z + H_{\text{ext}}]\mathbf{e}_z$, where A is the exchange constant, H_K is the anisotropy field, H_{ext} is the applied field, and $\mathbf{e}_z$ is the unit vector along the z direction.

When the uniaxial anisotropy field is larger than the demagnetization field, namely, $Q>1$ (This is the case for some metallic magnetic films[19] and multilayers[20]), we can get dark magnetic solitons for the m_z component. After rescaling the time coordinate and space coordinate by characteristic time $t_0 = 1/[\gamma(H_K - 4\pi M_s)]$ and length $l_0 = \sqrt{2A/[(H_K - 4\pi M_s)M_s]}$ and taking $\mathbf{M} = M_s\mathbf{m}$, the LLG equation can be simplified as

$$\frac{\partial \mathbf{m}}{\partial t} = -\left(\mathbf{m} \times \frac{\partial^2 \mathbf{m}}{\partial x^2}\right) - \left(m_z + \frac{H_{\text{ext}}}{H_K - 4\pi M_s}\right)(\mathbf{m} \times \mathbf{e_z}) + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} + \frac{b_J t_0}{l_0}\frac{\partial \mathbf{m}}{\partial x}. \tag{2}$$

Considering the fact that the magnitude of the magnetization is a constant at temperatures well below the Curie temperature, namely, $\mathbf{m}^2 = (\mathbf{M}/M_s)^2 = 1$, it is reasonable to take stereographic transformation,[18] $\psi = (m_x + i m_y)/(1 + m_z)$. Then, the equation about the complex function ψ is obtained,

$$i(1 - i\alpha)\frac{\partial \psi}{\partial t} = \frac{\partial^2 \psi}{\partial x^2} - 2\frac{\psi^*}{1 + \psi\psi^*}\left(\frac{\partial \psi}{\partial x}\right)^2 - \frac{H_{\text{ext}}}{H_K - 4\pi M_s}\psi - \frac{1 - \psi\psi^*}{1 + \psi\psi^*}\psi + i\frac{b_J t_0}{l_0}\frac{\partial \psi}{\partial x}. \tag{3}$$

We will get linear and nonlinear dynamics of magnetization from ψ.

As shown by Li and Zhang,[11] the spin-wave solution of the LLG equation is obtained in the presence of spin-polarized current. In the high-magnetic field, the deviation of magnetization from the the direction of the field is small. So, we only consider low-energy excitation. We take the spin-wave ansatz

$$\mathbf{m} = (\delta m_x \mathbf{e}_x + \delta m_y \mathbf{e}_y)e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)} + \mathbf{e}_z, \tag{4}$$

corresponding to $\psi = \psi_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$, where $\psi_0 = (\delta m_x + i\delta m_y)/2$, and δm_x and δm_y represent the rescaled amplitude of the spin waves in the x and y directions. Inserting ψ into Eq. (3) and keeping the linear terms in ψ_0, we obtain the dispersion relation of the spin wave

$$\omega = (1 + i\alpha)\omega_0[1 + l_0'^2(k_x - b_J M_s/\gamma A)^2], \tag{5}$$

where

$$\omega_0 = [\gamma(H_K - 4\pi M_s + H_{\text{ext}}) - b_J^2 M_s/2\gamma A]/(1 + \alpha^2),$$
$$l_0'^2 = 16\gamma^2 A^2/[8\gamma^2 M_s A(H_K - 4\pi M_s + H_{\text{ext}}) - M_s^2 b_J^2]. \tag{6}$$

It is easy to see that the spin-polarized current changes the energy gap. For large current density, $j_e \geqslant e/(\hbar P)\sqrt{2AM_s(H_K - 4\pi M_s + H_{\text{ext}})}$, the gap vanishes. This means that without other stimulation, such as thermal effects, the spin-polarized current excites spin waves when its density increases beyond a critical value. The Gilbert damping gives the energy gap a rescaling and damps the spin wave.

Now we investigate the nonlinear excitation of LLG equations. Considering small deviations of magnetization from the equilibrium direction (along the anisotropy axis) corresponding to $m_x^2 + m_y^2 \ll m_z^2$, or, $|\psi|^2 \ll 1$, and taking the long-wavelength approximation,[21] where $l_0 k \ll 1$, and k is the wave vector of spin wave that constitutes nonlinear excitations, Eq. (3) can be simplified, by keeping only the nonlinear terms of the order of the magnitude of $|\psi|^2\psi$. Without damping, we obtain the following nonlinear Schrödinger equation

$$i\frac{\partial \psi}{\partial t} = \frac{\partial^2 \psi}{\partial x^2} - \left(1 + \frac{H_{\text{ext}}}{H_K - 4\pi M_s}\right)\psi + 2|\psi|^2\psi + ib_J\frac{t_0}{l_0}\frac{\partial \psi}{\partial x}. \tag{7}$$

By the Hirota bilinear method,[22] the one- and two-soliton solutions of Eq. (7) are easily obtained. The one-soliton solution is $\psi = k_R e^{i[\eta_I + (1 + H_{\text{ext}}/(H_K - 4\pi M_s))t]}/\cosh[\eta_R + C/2]$, where $\eta = k[x + (b_J t_0/l_0 - ik)t]$, $e^C = e^{2|\lambda|}/(4k_R^2)$, the subscripts R and I denote the real and imaginary parts, and k is an arbitrary complex parameter and can be determined by the initial soliton location and amplitude. From ψ, we obtain the solution of magnetization,

$$m_x = \frac{2k_R}{\Delta}\cos\left[\eta_I + \left(1 + \frac{H_{\text{ext}}}{H_K - 4\pi M_s}\right)t\right]\cosh\left[\eta_R + \frac{C}{2}\right],$$
$$m_y = \frac{2k_R}{\Delta}\sin\left[\eta_I + \left(1 + \frac{H_{\text{ext}}}{H_K - 4\pi M_s}\right)t\right]\cosh\left[\eta_R + \frac{C}{2}\right],$$
$$m_z = \frac{1}{\Delta}\left[\cosh^2\left(\eta_R + \frac{C}{2}\right) - k_R^2\right], \tag{8}$$

where $\Delta = \cosh^2(\eta_R + C/2) + k_R^2$.

This one-soliton solution represents a 2π domain wall which is the bound state of a double π wall.[23] The height of the soliton is the rescaled magnetization. The velocity the of wall is $V = -b_J - 2k_I l_0/t_0$. As we see it, the spin-polarized current alters the soliton velocity as $-b_J$, proportional to the electron current density. The external fields induce the m_x and m_y components of the soliton to oscillate periodically. If applying alternating currents, $b_J = P j_e \mu_B/(eM_s)\cos(\omega t)$, and the only change to the solutions is $\eta = k[x + b_J/(\omega l_0)\sin(\omega t_0 t) - ikt]$. The center of the mass of solitons vibrate with the same frequency as ac currents. Inversely, we consider the effect of nonlinear excitations on the spin-polarized current. In ferromagnetic material with nonuniform magnetizations and adiabatic approximation, the spin-current density is $\mathbf{j}_x = (\mu_B/e)P j_e \mathbf{m}$[11]. Thus, the one-soliton excitation in nanowire gives spin current a magnetic pulse like the optical pulse in nonlinear media. In the same way, we can get the two-soliton solutions $\psi = (g_1 + g_3)/(1 + f_2 + f_4)e^{i[1 + H_{\text{ext}}/(H_K - 4\pi M_s)]t}$, and nonlinear dynamics of magnetization,

$$m_x = \frac{\psi + \psi^*}{1 + \psi\psi^*}, \quad m_y = -i\frac{\psi - \psi^*}{1 + \psi\psi^*}, \quad m_z = \frac{1 - \psi\psi^*}{1 + \psi\psi^*}, \tag{9}$$

where

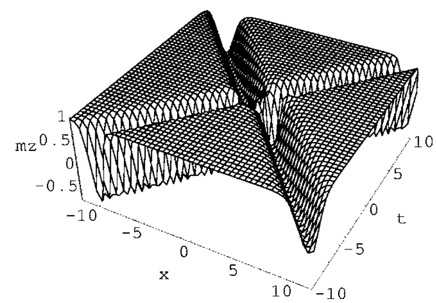

FIG. 2. The elastic collision of two dark solitons of m_z component where $k_1=1+0.5i$, $k_2=2-0.5i$, $\lambda_1=0.4+2i$, $\lambda_2=2i$.

$$g_1=\lambda_1 e^{\eta_1}+\lambda_2 e^{\eta_2},$$

$$g_3=e^{\eta_1+\eta_1^*+\eta_2+\theta_1}+e^{\eta_1+\eta_2^*+\eta_2+\theta_2},$$

$$f_2=e^{\eta_1+\eta_1^*+R_1}+e^{\eta_2+\eta_2^*+R_2}+e^{\eta_1+\eta_2^*+\delta}+e^{\eta_1^*+\eta_2+\delta^*},$$

$$f_4=e^{\eta_1+\eta_1^*+\eta_2^*+\eta_2+R_3}, \tag{10}$$

with

$$e^{\theta_1}=\frac{|\lambda_1|^2\lambda_2(k_1-k_2)^2}{(k_1+k_1^*)^2(k_1^*+k_2)^2},$$

$$e^{\theta_2}=\frac{|\lambda_2|^2\lambda_1(k_1-k_2)^2}{(k_2+k_2^*)^2(k_2^*+k_1)^2},$$

$$e^{R_1}=\frac{|\lambda_1|^2}{(k_1+k_1^*)^2},$$

$$e^{R_2}=\frac{|\lambda_2|^2}{(k_2+k_2^*)^2},$$

$$e^{\delta}=\frac{\lambda_1\lambda_2^*}{(k_1+k_2^*)^2},$$

$$e^{R_3}=\frac{|\lambda_1|^2|\lambda_2|^2|k_1^*-k_2^*|^4}{(k_1+k_1^*)^2(k_2+k_2^*)^2|k_1+k_2^*|^4}, \tag{10}$$

and $\eta_i=k_i[x+(b_Jt_0/l_0-ik_i)t]$, k_1, k_2, λ_1, and λ_2 are four complex parameters and determined by the initial conditions. In the limits $t\to\pm\infty$, the two-soliton solution is reduced to two single solitons similar to Eq. (8). That is to say, the two-soliton solution is combined with two one-solitons asymptotically. Asymptotical analysis indicates that the two solitons collide without amplitude switching, but the associate phase shift is $\delta\phi=(R_3-R_2-R_1)/2$, namely, the center of the solitons displace with $\delta\phi$. We show this two-soliton solution in Fig. 2, where we use the materials parameters of $CoPt_3$: $H_K=33778$ Oe, $A=1.0\times10^{-6}$ erg/cm, $M_s=1125$ G, $\gamma=1.75\times10^7$ Oe^{-1} s^{-1}, $P=0.35$, and take $H_{\text{ext}}=2000$ Oe, $j_e=10^6$ A/cm^2. So, we get $t_0=2.91\times10^{-12}$ s, $l_0=3.01\times10^{-7}$ cm, $b_J=18$ cm/s. The amplitudes and widths of m_x and m_y vary periodically with time because of the oscillating factor induced by the external and anisotropic fields. The two-soliton excitations bring the spin current two magnetic pulses with different velocities.

III. LOW-Q CASE

When $Q<1$, such as in permalloy, the nonlinear solutions of Eq. (1) are different. In this case, the characteristic time and length are $t_0=1/[\gamma(4\pi M_s-H_K)]$ and $l_0=\sqrt{2A/[(4\pi M_s-H_K)M_s]}$. By the same method, the LLG equation (1) is transformed into a nonlinear Schrödinger equation

$$i\frac{\partial\psi}{\partial t}=\frac{\partial^2\psi}{\partial x^2}+\left(1-\frac{H_{\text{ext}}}{4\pi M_s-H_K}\right)\psi-2|\psi|^2\psi+ib_J\frac{t_0}{l_0}\frac{\partial\psi}{\partial x}. \tag{12}$$

The one-soliton solution is $\psi=-\lambda e^{i\Psi}[(1+e^{i\Phi})+(1-e^{i\Phi})\tanh(\zeta/2)]$, where $\Psi=kx+[2\lambda^2+k^2+kb_Jt_0/l_0+1+H_{\text{ext}}/(4\pi M_s-H_K)]t+\Psi_0$, $\Phi=\arctan[P\sqrt{4\lambda^2-P^2}/(P^2-2\lambda^2)]$, and $\zeta=P[x-(\sqrt{4\lambda^2-P^2}-2k+b_Jt_0/l_0)t]+\zeta_0$, in which k, λ, P, Ψ_0, and ζ_0 are real constants. So, the solutions of magnetization are

$$m_x=\frac{\lambda}{\Lambda}[\cos(\Phi+\Psi)-\cos\Psi+[\cos(\Psi+\Phi)+\cos\Psi]\tanh(\zeta/2)],$$

$$m_y=\frac{\lambda}{\Lambda}[\sin(\Phi+\Psi)-\sin\Psi+[\sin(\Psi+\Phi)+\sin\Psi]\tanh(\zeta/2)],$$

$$m_z=\frac{1}{4\Lambda}[4(1-\lambda^2)+P^2-P^2\tanh^2(\zeta/2)], \tag{13}$$

where $\Lambda=[4(1+\lambda^2)-P^2+P^2\tanh^2(\zeta/2)]/4$. In this solutions, in addition to the change of velocity, spin-polarized current induces the oscillation of the m_x and m_y components.

Also, it is easy to find the two-soliton solutions of Eq. (12) $\psi=-\lambda e^{i\Psi}(1+g_1+g_2)/(1+f_1+f_2)$, and the nonlinear evolution of magnetization like that of Eq. (9). The functions in the solution are as follows:

$$g_1=e^{i\Phi_1}\exp\zeta_1+e^{i\Phi_2}\exp\zeta_2,$$

$$g_2=Ce^{i(\Phi_1+\Phi_2)}\exp(\zeta_1+\zeta_2),$$

$$f_1=\exp\zeta_1+\exp\zeta_2,$$

$$f_2=A\exp(\zeta_1+\zeta_2),$$

where

$$\zeta_j=P_j[x-(\sqrt{4\lambda^2-P_j^2}-2k+kb_Jt_0/l_0)t]+\zeta_{j0},$$

$$\Phi_j=\arctan[P_j\sqrt{4\lambda^2-P_j^2}/(P_j^2-2\lambda^2)],\quad j=1,2,$$

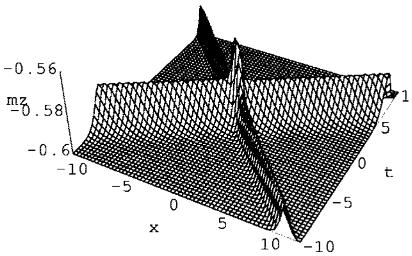

FIG. 3. The elastic collision of two bright solitons of an m_z component where $P_1=1.2$, $P_2=3.9$, $\lambda=2$, $k=1$, $\Psi_0=2$, $\zeta_{10}=1$, $\zeta_{20}=3$.

$$C=\frac{(P_1-P_2)^2+(\sqrt{4\lambda^2-P_1^2}-\sqrt{4\lambda^2-P_2^2})^2}{(P_1+P_2)^2+(\sqrt{4\lambda^2-P_1^2}-\sqrt{4\lambda^2-P_2^2})^2},$$

with P_1, P_2, ζ_{10}, and ζ_{20} are real constants. Using asymptotical analysis, we find that the two solitons collide with phase shift $\delta\phi=\ln C$. After the collision, the centers of solitons displace with $\ln A$. In Fig. 3, we plot this two-soliton solution with material parameters of permalloy: $H_K=10$ Oe, $A=1.3\times10^{-6}$erg/cm, $M_s=800$ G, $P=0.5$ and $H_{\mathrm{ext}}=2000$ Oe, $j_e=10^6$ A/cm^2. The characteristic time and length is $t_0=4.55\times10^{-11}$s, $l_0=1.14\times10^{-6}$cm, and $b_J=51$ cm/s.

When the external magnetic field deviates from the z axis, in general, the equations cannot be exactly solved.[24] However, some special solutions can be found. For example, we consider the case that $\mathbf{H}_{\mathrm{ext}}=H_x\mathbf{e}_x+H_z\mathbf{e}_z$ and $Q<1$. In the presence of spin-polarized current, there exists a dynamic soliton solution

$$m_x=\frac{1-D^2\cosh^2\xi}{1+D^2\cosh^2\xi}\cos(\gamma H_z t_0 t),$$

$$m_y=\frac{1-D^2\cosh^2\xi}{1+D^2\cosh^2\xi}\sin(\gamma H_z t_0 t),$$

$$m_z=\frac{2D\cosh\xi}{1+D^2\cosh^2\xi},$$

where $\xi=\sqrt{1-\gamma t_0 H_x}(x+b_J t_0/l_0 t)$, $D=\sqrt{(\gamma H_x t_0)/(1-\gamma H_x t_0)}$. This solution is plotted in Fig. 4 with the same material parameters as above. The depth and width of the trough decrease as H_x increases. The amplitudes of m_x and m_y oscillate with frequency γH_z. According to this solution, when the external field perpendicular to wire's axis vanishes, the magnetization rotates purely in a plane parallel to the wire's axis.

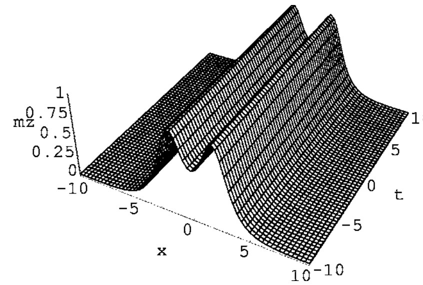

FIG. 4. Two neighbor bright solitons of m_z component with $H_y=1000$ Oe.

IV. CONCLUSION

Using a stereographic projection, we transform the modified LLG equation with a spin torque into a nonlinear equation of complex function. By the Hirota bilinear method, we get one- and two-soliton solutions describing nonlinear magnetization in ferromagnetic nanowire with spin-polarized current, and these solitons propagate along the wire's axis. These results indicate that the soliton velocity varies due to the spin torque, the change is proportional to current density, and the amplitudes are modulated by spin-polarized current. The centers of the solitons vibrate periodically with the frequency of alternating currents. In the case of multisoliton solutions, magnetic solitons collide elastically with phase shift.

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