Exact soliton solution and inelastic two-soliton collision in a spin chain driven by a time-dependent magnetic field

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We investigate dynamics of exact *N*-soliton trains in a spin chain driven by a time-dependent magnetic field by means of an inverse scattering transformation. The one-soliton solution indicates obviously the spin precession around the magnetic field and periodic shape variation induced by the time-varying field as well. In terms of the general soliton solutions, *N*-soliton interaction and particularly various two-soliton collisions are analyzed. The inelastic collision by which we mean the soliton shape change before and after collision appears is generally due to the time-varying field. We, moreover, show that complete inelastic collisions can be achieved by adjusting spectrum and field parameters. This may lead to a potential technique of shape control of soliton.

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I. INTRODUCTION

Over the past three decades, an enormous amount of literature has appeared throughout soliton physics and the underlying completely integrable models. The classical Heisenberg spin chain which exhibits both coherent and chaotic structures depending on the nature of the magnetic interactions [1–4] has attracted considerable attentions in nonlinear science and condensed-matter physics. Solitons in quasi-one-dimensional magnetic systems have already been probed experimentally by neutron inelastic scattering [5,6], nuclear magnetic resonance [7,8], Mossbauer linewidth measurements [9], and electron spin resonance [10]. The corresponding theoretical studies are based usually on the Landau-Lifshitz equation [11]. The isotropic spin chain has been studied in various aspects [12–16], and the construction of soliton solutions of Landau-Lifschitz equation with an easy axis has been also discussed [17,18]. It is demonstrated that the inverse scattering transformation [14,19–21] can be used to solve the Landau-Lifschitz equation for an anisotropic spin chain. Great efforts [22,23] have been devoted to construct the soliton solution which is found by means of the Darboux transformation [24]. The continuum spin chain in an external magnetic field is of great interest and multisoliton solutions of Landau-Lifschitz equation for an isotropic spin chain have been reported [25]. Using Darboux transformation, the nonlinear dynamics of anisotropic Heisenberg spin chain in an external magnetic field is investigated and exact soliton solutions are obtained [26]. Recently soliton interaction has been investigated [16]. The main goal of this paper is to study the new effect of soliton-soliton interaction in a spin chain driven by time oscillating magnetic field. We obtain exact solution of *N*-soliton trains in terms of an inverse scattering transformation. It is shown that inelastic collisions generally appear due to the time-varying field and the complete inelastic collisions which may lead to an interesting technique of soliton filter and switch can be achieved in a special case.

The outline of this paper is organized as follows. In Sec. II, the formalism obtained by an inverse scattering transformation is explained in detail and the general *N*-soliton solution for reflectionless case is obtained. Precession of nonlinear spin waves in the oscillating magnetic field is shown in Sec. III. Section IV is devoted to general two-soliton solution and soliton collisions. Finally, Sec. V will give our concluding remarks.

II. EXACT SOLUTION OF *N*-SOLITON TRAIN

Our starting Hamiltonian describing the spin chain in a time oscillating magnetic field with an arbitrary direction can be written as

$$\hat{H}=-J\sum_{\langle n,n'\rangle}\hat{S}_n\cdot\hat{S}_{n'}-g\mu_B\mathbf{B}(t)\cdot\sum_n\hat{S}_n, \tag{1}$$

where $\hat{S}_n\equiv(\hat{S}_n^x,\hat{S}_n^y,\hat{S}_n^z)$ with $n=1,2,\ldots,N$ are spin operators, $J>0$ is the pair interaction parameter, g is the Lande factor, μ_B is the Bohr magneton, and $\mathbf{B}(t)=B\cos(\omega t)\mathbf{e}$ is the external magnetic field with $\mathbf{e}=(\sin\theta,0,\cos\theta)$ denoting the unit vector of field direction, where chain axis and direction of magnetic field are assumed in x-z plane. The angle θ between direction of magnetic field and z axis is arbitrary.

The equation of motion for the spin operator on the nth site is $(d/dt)\hat{\mathbf{S}}_n=-(i/\hbar)[\hat{\mathbf{S}}_n,\hat{H}]$. At low temperatures, the spin can be treated as a classical vector such that $\hat{\mathbf{S}}_n\rightarrow\mathbf{S}(x)$. So that the equation of motion in a continuum spin chain under a time-dependent magnetic field can be obtained as a Landau-Lifschitz type:

$$\frac{\partial}{\partial t}\mathbf{S}=\mathbf{S}\times\left(\frac{\partial^2}{\partial x^2}\mathbf{S}+\boldsymbol{\varepsilon}\right), \tag{2}$$

with $\boldsymbol{\varepsilon}=g\mu_B\mathbf{B}(t)/(2J)$, where $\mathbf{S}(x,t)=(S^x(x,t),S^y(x,t),S^z(x,t))$. We set the length of the spin vector to unity for the sake of simplicity, $\mathbf{S}^2(x,t)=1$. The dimensionless time t and coordinate x in Eq. (2) are scaled in units $1/(2J)$ and d, respectively, where d denotes the lattice constant.

The corresponding Lax equations for the equation of motion (2) are written as

$$\frac{\partial}{\partial x}\Psi(x,t,\lambda)=L(\lambda)\Psi(x,t,\lambda),$$

$$\frac{\partial}{\partial t}\Psi(x,t,\lambda)=M(\lambda)\Psi(x,t,\lambda), \tag{3}$$

where λ is the spectral parameter, $\Psi(x,t,\lambda)$ is eigenfunction corresponding to λ, and L and M are given in the form

$$L=-i\lambda(\mathbf{S}\cdot\boldsymbol{\sigma}),$$

$$M=\frac{i}{2}(\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma})+i2\lambda^2(\mathbf{S}\cdot\boldsymbol{\sigma})-\lambda(\mathbf{S}\cdot\boldsymbol{\sigma})\left(\frac{\partial}{\partial x}\mathbf{S}\cdot\boldsymbol{\sigma}\right). \tag{4}$$

Here $\boldsymbol{\sigma}$ is Pauli matrix. Thus Eq. (2) can be recovered from the compatibility condition $(\partial/\partial t)L-(\partial/\partial x)M+[L,M]=0$. Based on the Lax equations (3), we derive the exact N-soliton solution by employing the inverse scattering transformation. We consider the following natural boundary condition of initial time $(t=0)$: $\mathbf{S}(x)\equiv(S^x,S^y,S^z)\rightarrow(\sin\theta,0,\cos\theta)$ as $|x|\rightarrow\infty$, namely, the spin vector is along the field direction. We then have the asymptotic form of Eq. (3) at $|x|\rightarrow\infty$,

$$\partial_x E(x,\lambda)=L_0(\lambda)E(x,\lambda), \tag{5}$$

where

$$E(x,\lambda)=Ue^{-i\lambda x\sigma_3},\quad L_0(\lambda)=-i\lambda U_0 \tag{6}$$

and

$$U_0=\begin{pmatrix}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{pmatrix},\quad U=\begin{pmatrix}1 & -\tan\frac{\theta}{2}\\ \tan\frac{\theta}{2} & 1\end{pmatrix}. \tag{7}$$

The Jost solutions $\Psi_+(x,\lambda)$ and $\Psi_-(x,\lambda)$ of Eq. (3) are defined as

$$\Psi_+(x,\lambda)\rightarrow E(x,\lambda)\quad \text{as}\ \ x\rightarrow\infty,$$

$$\Psi_-(x,\lambda)\rightarrow E(x,\lambda)\quad \text{as}\ \ x\rightarrow-\infty.$$

With standard procedures, one finds the following integral representations of the Jost solutions in terms of the integration kernels K and N to be determined:

$$\Psi_+(x,\lambda)=Ue^{-i\lambda x\sigma_3}+\lambda\int_x^\infty dyK(x,y)Ue^{-i\lambda y\sigma_3},$$

$$K(x,\infty)=0,\quad K(x,y)=0\quad \text{as}\ \ y<x \tag{8}$$

and

$$\Psi_-(x,\lambda)=Ue^{-i\lambda x\sigma_3}+\lambda\int_{-\infty}^x dyN(x,y)Ue^{-i\lambda y\sigma_3},$$

$$N(x,-\infty)=0,\quad N(x,y)=0\quad \text{as}\ \ y<x, \tag{9}$$

where K and N are 2×2 matrices. Substituting $\Psi_+(x,\lambda)$ of Eq. (8) into Eq. (3) and noting $U\sigma_3U^{-1}=U_0$, we obtain

$$\mathbf{S}\cdot\boldsymbol{\sigma}=[I-iK(x,x)U_0]U_0[I-iK(x,x)U_0]^{-1}, \tag{10}$$

where I is a unit matrix. It is obvious that Eq. (10) gives rise to a relation between kernel K and spin vector $\mathbf{S}$ to be obtained.

The scattering data for the operator $L(x,\lambda)$ are the set $s=\{a(\lambda),b(\lambda);\lambda_n,c_n,\mathrm{Im}\lambda>0,n=1,\ldots,N\}$, where $|a(\lambda)|^2+|b(\lambda)|^2=1$, and the function $a(\lambda)$ can be analytically continued to the half plane $\mathrm{Im}\lambda>0$. The discrete eigenvalues λ_n for the operator $L(x,\lambda)$ are zeros of $a(\lambda)$ such that $a(\lambda_n)=0$ (for simplicity we consider only simple zeros). The functions $a(\lambda)$ and $b(\lambda)$ are seen to be transmission and reflection coefficients of the operator L, respectively. The parameter c_n denotes the asymptotic characteristics of the eigenfunctions.

The time dependence of the scattering data $s(t)$ can be obtained from the second Lax equation (3),

$$a(\lambda,t)=a(\lambda,0),$$

$$b(\lambda,t)=\exp\left(-4i\lambda^2t-i\frac{g\mu_BB\sin\omega t}{J\omega}\right)b(\lambda,0),$$

$$\lambda_n(t)=\lambda_n(0),$$

$$c_n(t)=\exp\left(-4i\lambda_n^2t-i\frac{g\mu_BB\sin\omega t}{J\omega}\right)c_n(0), \tag{11}$$

where $c_n(0)$, $b(\lambda,0)$ and $a(\lambda,0)$, are constants determined by initial conditions. The Gelfand-Levitan-Marchenko equation establishes a relation between the kernel $K(x,y,t)$ and the scattering data $s(t)$ and has the form

$$K(x,y,t)U\begin{pmatrix}1\\0\end{pmatrix}+F_1+\frac{1}{2\pi}\int_{-\infty}^\infty\lambda^{-1}r(\lambda)F_2d\lambda=0, \tag{12}$$

as $y>x$, where $r(\lambda)=b(\lambda)/a(\lambda)$ and

$$F_1=U\begin{pmatrix}0\\1\end{pmatrix}\sum_{n=1}^N\frac{c_n(t)}{\lambda_n}e^{i\lambda_n(x+y)}+\int_x^\infty K(x,z,t)U\begin{pmatrix}0\\1\end{pmatrix}\sum_{n=1}^N c_n(t)e^{i\lambda_n(y+z)}dz,$$

$$F_2=U\begin{pmatrix}0\\1\end{pmatrix}e^{i\lambda x}+\lambda\int_x^\infty K(x,z,t)U\begin{pmatrix}0\\1\end{pmatrix}e^{i\lambda z}dz. \tag{13}$$

For the reflectionless case, $r(\lambda)=0$, Eq. (12) becomes a set of algebraic equations and after tedious calculation the matrix elements of the kernel K are obtained as

$$K_{11}(x,x,t)=\cos^2\frac{\theta}{2}\left[B_1+B_2\tan\frac{\theta}{2}\right],$$

$$K_{12}(x,x,t)=\cos^2\frac{\theta}{2}\left[B_1\tan\frac{\theta}{2}-B_2\right], \tag{14}$$

with

$$B_1=\frac{\det\left[I+G''G'+D^T\left(C\tan\frac{\theta}{2}-\bar{C}G'\right)\right]}{\det(I+G'G'')}-1,$$

$$B_2=\frac{\det\left[I+G'G''-\bar{D}^T\left(\bar{C}+CG''\tan\frac{\theta}{2}\right)\right]}{\det(I+G'G'')}-1, \tag{15}$$

where $C(x,t)$, $C'(x,t)$, $D(x)$ are $1\times N$ matrices and $G'(x,t)$, $G''(x,t)N\times N$ matrices, respectively. The superscript T means the transposed matrix and the overbar denotes complex conjugate,

$$C(x,t)_n=c_n(t)\lambda_n^{-1}D(x)_n,$$

$$C'(x,t)_n=c_n(t)D(x)_n,$$

$$D(x)_n=\exp(i\lambda_n x),$$

$$G'(x,t)_{nm}=\frac{1}{i(\bar{\lambda}_n-\lambda_m)}\overline{D(x)}_nC'(x,t)_m,$$

$$G''(x,t)_{nm}=\frac{1}{-i(\lambda_n-\bar{\lambda}_m)}D(x)_n\overline{C'(x,t)}_m. \tag{16}$$

Substituting Eq. (14) into Eq. (10), we obtain the general form of N-soliton trains,

$$S^x=\frac{1}{\Delta}\mathrm{Re}\{-i2K_{12}[1-iK_{11}\cos\theta]+[1+K_{11}^2-K_{12}^2]\sin\theta\},$$

$$S^y=\frac{-1}{\Delta}\mathrm{Im}\{-i2K_{12}[1-iK_{11}\cos\theta]+[1+K_{11}^2-K_{12}^2]\sin\theta\},$$

$$S^z=\frac{1}{\Delta}\{[1+|K_{11}|^2-|K_{12}|^2]\cos\theta$$
$$+2\mathrm{Im}[K_{11}(1+i\bar{K}_{12}\sin\theta)]\}, \tag{17}$$

where $\bar{K}_{12}$ is the complex conjugate of K_{12},

$$\Delta=|1-i[K_{11}\cos\theta+K_{12}\sin\theta]|^2+|K_{11}\sin\theta-K_{12}\cos\theta|^2. \tag{18}$$

According to exact N-soliton solutions in Eq. (17), we, generally speaking, can investigate the dynamics of soliton trains and soliton interaction. The neighboring solitons may repulse or attract each other with a force depending on their phase difference. Particularly we in the following shall concentrate on the analyses of one-soliton dynamics and two-soliton collisions which may be of more interest.

III. ONE-SOLITON DYNAMICS AND SPIN PRECESSION WITH TIME-VARYING AMPLITUDE

When $N=1$, from Eqs. (14) and (17), we obtain the general form of the exact one-soliton solution as follows:

$$S^x=\frac{R_1}{|\lambda_1|^4\cosh^2\Theta_1},$$

$$S^y=\frac{R_2}{|\lambda_1|^4\cosh^2\Theta_1},$$

$$S^z=R_3\cos\theta+R_4\sin\theta, \tag{19}$$

where

$$\begin{aligned}R_1=&[|\lambda_1|^4\cosh^2\Theta_1+\beta_1^2(\alpha_1^2-\beta_1^2\cos 2\theta)e^{-2\Theta_1}]\sin\theta\\&+\beta_1^2|\lambda_1|^2(2\cos^2\theta\sin^2\Phi_1-1)\sin\theta\\&+2\beta_1^2|\lambda_1|(2\beta_1\sin\Phi_1\sin^2\theta+\alpha_1\cos\Phi_1)e^{-\Theta_1}\cos\theta\\&-2\beta_1(\beta_1e^{-\Theta_1}\sin\theta+|\lambda_1|\sin\Phi_1\cos\theta)[|\lambda_1|^2\cosh\Theta_1\\&+\beta_1(|\lambda_1|\sin\Phi_1\sin\theta-\beta_1e^{-\Theta_1}\cos\theta)\cos\theta],\end{aligned}$$

$$\begin{aligned}R_2=&2\alpha_1\beta_1^2|\lambda_1|e^{-\Theta_1}\sin\Phi_1+2\beta_1|\lambda_1|^3\cos\Phi_1\cosh\Theta_1\\&-2\beta_1^3|\lambda_1|(\sin\theta+\cos^2\theta)e^{-\Theta_1}\cos\Phi_1,\end{aligned}$$

$$R_3=1-\frac{2\beta_1^2}{|\lambda_1|^2\cosh^2\Theta_1},$$

$$\begin{aligned}R_4=&\frac{1}{|\lambda_1|^2\cosh^2\Theta_1}[2\beta_1^2\cos(\Phi_1-\phi_1)\sinh\Theta_1\\&+2\alpha_1\beta_1\sin(\Phi_1-\phi_1)\cosh\Theta_1],\end{aligned} \tag{20}$$

with

$$\Theta_1=2\beta_1(x-V_1t)-x_1,$$

$$V_1=4\alpha_1,\quad x_1=\ln[c_1(0)/(2\beta_1)],$$

$$\Phi_1=2\alpha_1x-4(\alpha_1^2-\beta_1^2)t-\frac{g\mu_BB}{\omega J}\sin(\omega t)-\phi_1,$$

$$\Omega_1=[2(\alpha_1^2-\beta_1^2)]/\alpha_1+\Omega_B,$$

$$\Omega_B=[g\mu_BB\cos(\omega t)]/(2\alpha_1J), \tag{21}$$

here $\phi_1=\arg\lambda_1$ and $\lambda_1=\alpha_1+i\beta_1$ is the eigenvalue parameter. Solution (19) describes a spin precession around magnetic field direction characterized by four real parameters: velocity V_1, frequency Ω_1, coordinate of the center of the solitary wave x_1, and initial phase ϕ_1. The center of solitary wave moves with a velocity V_1, while the wave depth and width vary periodically with time. The wave shape is modulated periodically by frequency Ω_1 depending on magnetic field. Therefore, solution (19) cannot be written as the form of separating variables. Amplitude A and phase Φ_1 are complicated functions of J, B, ω, and λ_1. When $\alpha_1=\beta_1$, the frequency Ω_1 depends on magnetic field only, and we have $\Omega_1=\Omega_B$. If $\alpha_1=\beta_1$ and $B=0$, solution (19) reduces to the usual soliton without shape changing. Therefore, we can use magnetic field to adjust spin precession and the wave shape as well.

For a special case, $\theta=0$, namely the magnetic field is along the z-axis, S^z is independent of magnetic field, $S^z=R_3$, while S^x and S^y precess around the magnetic field (z axis). The precession frequency Ω_1 is determined by magnetic field. As the magnetic field rotates from $\theta=0$ (z axis) to $\theta=\pi/2$ (x axis), we can find the correspondence such that $S^x\rightarrow -S^z$, $S^y\rightarrow S^y$, $S^z\rightarrow S^x$. The three components of spin vector satisfy "left-hand rule." When $\theta=\pi/2$, S^x is independent of magnetic field, while S^y and S^z precess around the magnetic field (x axis). These results show that the magnetic field results in the motion of the center of solitary waves along the field direction and the spin vector rotates around the field in any case.

IV. TWO-SOLITON COLLISION

When $N=2$, from Eqs. (14) and (17), the general form of the exact two-soliton solution is seen to be

$$S^x=\mathrm{Re}[-i2Q_2(1-iQ_1\cos\theta)+(1+Q_1^2-Q_2^2)\sin\theta],$$

$$S^y=\mathrm{Im}[i2Q_2(1-iQ_1\cos\theta)-(1+Q_1^2-Q_2^2)\sin\theta],$$

$$S^z=(1+|Q_1|^2-|Q_2|^2)\cos\theta+2\,\mathrm{Im}[Q_1(1+i\overline{Q}_2\sin\theta)], \tag{22}$$

where

$$Q_1=\frac{\cos^2\frac{\theta}{2}}{W}\Big\{(f_1-f_3)f_6+(f_2-f_4)f_5+\tan\frac{\theta}{2}[(\overline{f_1}-\overline{f_3})f_8+(\overline{f_2}-\overline{f_4})f_7]\Big\},$$

$$Q_2=\cos^2\frac{\theta}{2}W\Big\{[(f_1-f_3)f_6+(f_2-f_4)f_5]\tan\frac{\theta}{2}-(\overline{f_1}-\overline{f_3})f_8-(\overline{f_2}-\overline{f_4})f_7\Big\}, \tag{23}$$

with

$$f_1=1+|q_1|^2+\chi_1\overline{\chi_2}q_1\overline{q_2},$$

$$f_2=1+|q_2|^2+\overline{\chi_1}\chi_2\overline{q_1}q_2,$$

$$f_3=\overline{\chi_1}|q_1|^2+\chi_1q_1\overline{q_2},$$

$$f_4=\overline{\chi_2}|q_2|^2+\chi_2\overline{q_1}q_2,$$

$$f_5=\xi_1\left(q_1\tan\frac{\theta}{2}-|q_1|^2\right)-\chi_1\xi_2q_1\overline{q_2},$$

$$f_6=\xi_2\left(q_2\tan\frac{\theta}{2}-|q_2|^2\right)-\chi_2\xi_1\overline{q_1}q_2,$$

$$f_7=-\xi_1\left(\overline{q_1}+|q_1|^2\tan\frac{\theta}{2}\right)-\xi_2\overline{\chi_1}\overline{q_1}q_2\tan\frac{\theta}{2},$$

$$f_8=-\xi_2\left(\overline{q_2}+|q_2|^2\tan\frac{\theta}{2}\right)-\xi_1\overline{\chi_2}q_1\overline{q_2}\tan\frac{\theta}{2},$$

$$\chi_1=\frac{2\beta_1\lambda_1}{-i(\lambda_1-\overline{\lambda_2})|\lambda_1|},\quad \chi_2=\frac{2\beta_2\lambda_2}{-i(\lambda_2-\overline{\lambda_1})|\lambda_2|},$$

$$W=f_1f_2-f_3f_4,$$

$$q_j=e^{-\Theta_j+i\Phi_j},$$

$$\xi_j=2\beta_j|\lambda_j|^{-1}, \tag{24}$$

and

$$\Theta_j=2\beta_j(x-V_jt)-x_j,$$

$$V_j=4\alpha_j,\quad x_j=\ln[c_j(0)/(2\beta_j)],$$

$$\Phi_j=2\alpha_jx-4(\alpha_j^2-\beta_j^2)t-\frac{g\mu_BB}{\omega J}\sin(\omega t)-\phi_j,$$

$$\Omega_j=[2(\alpha_j^2-\beta_j^2)]/\alpha_j+\Omega_B,$$

$$\Omega_B=[g\mu_BB\cos(\omega t)]/(2\alpha_jJ), \tag{25}$$

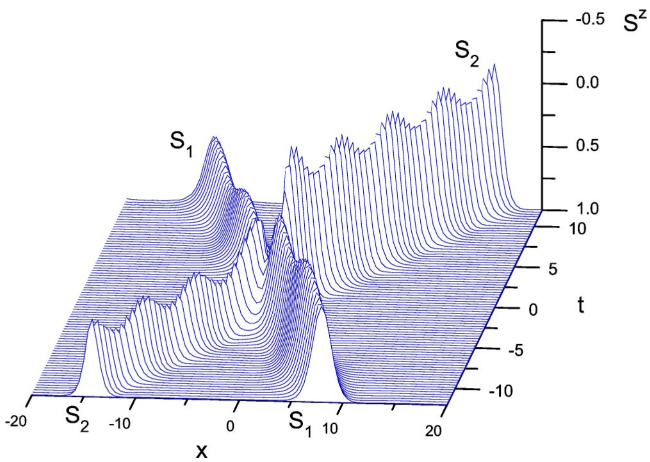

FIG. 1. Inelastic head on collision between two solitons—profiles of z component $S^z(x,t)$ of spin vector in Eq. (22) in a spin chain under a time-dependent magnetic field showing two different dramatic scenarios of the shape changing collision, where $\theta=\pi/36$, $\lambda_1=-0.2+i0.45$, $\lambda_2=0.3+i0.65$, $c_1(0)=0.2$, $c_2(0)=3.5$, $(g\mu_BB)/J=0.01$, $\omega=10$, $V_1=-0.8$, $V_2=1.2$. All quantities plotted are dimensionless.

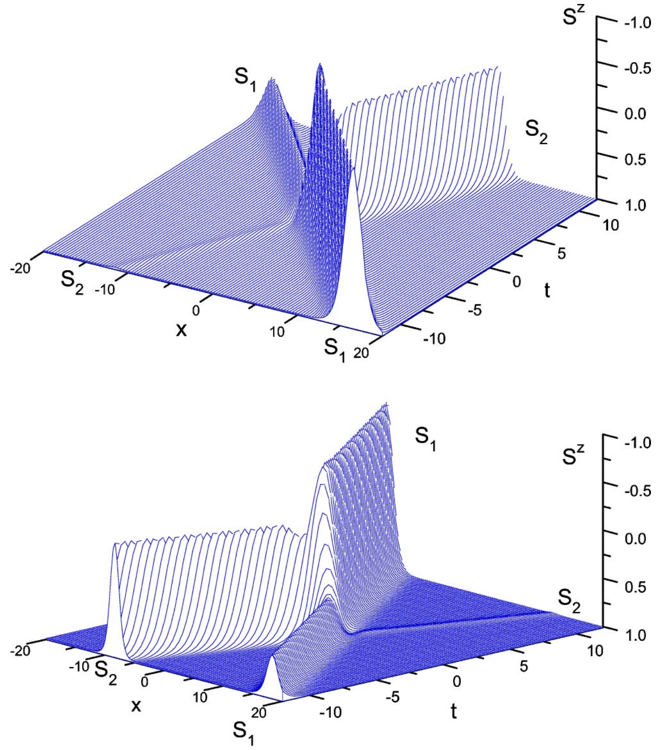

FIG. 2. (a) Complete inelastic head on collision expressed by Eq. (22) when S_1 suppressed, where $\theta=0$, $\lambda_1=-0.35+i0.4$, $\lambda_2=0.2-i0.6$, $c_1(0)=0.2$, $c_2(0)=2.5$, $(g\mu_B B)/J=0.01$, $\omega=10$, $V_1=-1.4$, $V_2=0.8$. All quantities plotted are dimensionless. (b) Complete inelastic head on collision expressed by Eq. (22) when S_2 suppressed, where $\theta=0$, $\lambda_1=-0.35-i0.4$, $\lambda_2=0.2+i0.6$, $c_1(0)=0.2$, $c_2(0)=2.5$, $(g\mu_B B)/J=0.01$, $\omega=10$, $V_1=-1.4$, $V_2=0.8$. All quantities plotted are dimensionless.

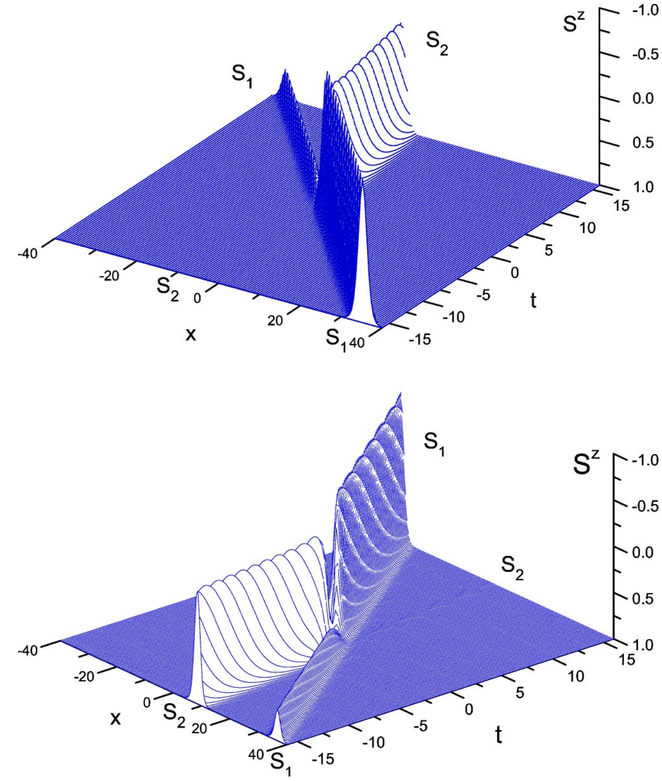

FIG. 3. (a) Complete inelastic overtake collision expressed by Eq. (22) when S_1 suppressed, where $\theta=0$, $\lambda_1=-0.55+i0.4$, $\lambda_2=-0.1-i0.45$, $c_1(0)=0.2$, $c_2(0)=2.5$, $(g\mu_B B)/J=0.01$, $\omega=10$, $V_1=-2.2$, $V_2=-0.4$. All quantities plotted are dimensionless. (b) Complete inelastic overtake collision expressed by Eq. (22) when S_1 suppressed, where $\theta=0$, $\lambda_1=-0.55-i0.4$, $\lambda_2=-0.1+i0.45$, $c_1(0)=0.2$, $c_2(0)=2.5$, $(g\mu_B B)/J=0.01$, $\omega=10$, $V_1=-2.2$, $V_2=-0.4$. All quantities plotted are dimensionless.

here $\phi_j=\arg\lambda_j$ and $\lambda_j=\alpha_j+i\beta_j$ is the eigenvalue parameter, $j=1,2$. Solutions (22) describe a general inelastic scattering process of two solitary waves with different center velocities V_1 and V_2, different shape-variation frequencies Ω_1 and Ω_2. Before collision, they move towards each other, one with velocity V_1 and shape-variation frequency Ω_1, the other with V_2 and Ω_2. The interaction potential between two solitons is a complicated function of parameters J,B,ω, and λ_j. When $\alpha_j=\beta_j$, two-soliton shape-variation frequencies $\Omega_j(j=1,2)$ are determined by magnetic field. In the case of $B=0$, solutions (22) reduce to that of the usual two soliton with two center velocities while without a change in shape, where an interesting process in the absence of magnetic field is that the collision can result in the interchange of amplitude A_j and phase $\Phi_j(j=1,2)$ like exactly in the case of elastic collision of two particles.

In order to understand the nature of two-soliton interaction, we analyze asymptotic behavior of two-soliton solutions (22). Asymptotically, the two-soliton waves (22) can be written as a combination of two one-soliton waves (19) with different amplitude and phase. The formation of two-soliton waves in the corresponding limits $x\rightarrow-\infty$ and $x\rightarrow\infty$ is similar to that of one-soliton waves (19). Analysis reveals that there is an amplitude exchange among three components S^x, S^y, and S^z of each soliton during collision, which can be described by a transition matrix T_l^k such that $A_l^{k+}=A_l^{k-}T_l^k$, where the subscript $l=1,2$, respectively, represents the first and the second soliton, $k=x,y,z$ denote three components of each soliton, and the sign $\pm$ denotes the asymptotic limits of the corresponding amplitude $A_l^{k\pm}$, at $x\rightarrow\pm\infty$. As a consequence, change in amplitude of the three components S_1^k of the first soliton from A_1^{k-} to A_1^{k+} is given by square of transition matrices $|T_1^k|^2$ along with phase shift $\delta\Phi_1^k$ during collision. In a similar fashion, the three components S_2^k of the second soliton also change amplitudes from A_2^{k-} to A_2^{k+} with a quantity $|T_2^k|^2$. The associate phase shift for the second soliton is $\delta\Phi_2^k$. We also note a net change of separation distance between two solitons by δX_{12}.

For the special case $|T_l^k|=1$, which is possible only when $\lambda_2=-\overline{\lambda}_1$, we have the standard elastic collision. For all other cases, we have the quantity $|T_l^k|\neq1$, which corresponds to the relative change among three components of the spin vector leading to the deformation of soliton shape. However, the total amplitude of individual solitons S_1 and S_2 is a conserved quantity, i.e., $\Sigma_l|A_l^{k\pm}|^2$ is constant for $l=1,2$.

It is interesting to show the inelastic collision graphically. The general inelastic head on collision is explained in Fig. 1

from which it is seen that the amplitudes of S_1, S_2 are, respectively, suppressed and enhanced after collision. Figure 2 is devoted to the complete inelastic head on collisions. The amplitudes of S_1 and S_2 are, respectively, suppressed after the collision shown in Figs. 2(a) and 2(b). The complete inelastic overtake collision is shown in Fig. 3 with the amplitudes of $S_1$and S_2 suppressed, respectively.

V. CONCLUSION

In terms of an inverse scattering transformation, the exact solution of N-soliton trains in a spin chain driven by a time oscillating magnetic field is obtained. From the general solution, the dynamics and soliton interactions are analyzed. The one-soliton solution gives rise explicitly to the spin precession along with the soliton shape variation induced by the time-varying field. It is also shown that the time-varying field leads generally to the inelastic and particularly the complete inelastic two-soliton collisions, which may be useful in developing a soliton-shape control technique.

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