Soliton solution for the spin current in a ferromagnetic nanowire

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We investigate the interaction of a periodic solution and a one-soliton solution for the spin-polarized current in a uniaxial ferromagnetic nanowire. The amplitude and wave number of the periodic solution for the spin current give different contributions to the width, velocity, and amplitude of the soliton. Moreover, we found that the soliton can be trapped only in space with proper conditions. Finally, we analyze the modulation instability and discuss dark solitary wave propagation for a spin current on the background of a periodic solution. In some special cases, the solution can be expressed as the linear combination of the periodic and soliton solutions.

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I. INTRODUCTION

The study of magnetoelectronics has received considerable interest for its potential technological applications. Theoretical and experimental investigations, mainly concentrated on giant magnetoresistance, are of fundamental importance in the understanding of magnetism, and may be applied in the fabrication of magnetic devices. In metallic ferromagnets, the differences between the electronic bands and the scattering cross sections of impurities for majority and minority spins at the Fermi energy cause spin-dependent mobilities [1]. The difference between spin-up and spin-down electric currents is called the spin current. It is a tensor, with a direction of flow and spin polarization parallel to the equilibrium magnetization vector $\mathbf{M}(\mathbf{r},t)$ [2],

$$\mathcal{J}=\frac{P\mu_B}{eM_s}\mathbf{j}_e\otimes\mathbf{M}(\mathbf{r},t), \tag{1}$$

where P is the spin polarization of the current, μ_B is the Bohr magneton, and e is the magnitude of the electronic charge. The vector $\mathbf{j}_e$ tracks the direction of the charge current, $\mathbf{M}(\mathbf{r},t)$ describes the direction of the spin polarization of the current, and M_s is the saturation magnetization.

When the magnetization directions in the systems are not collinear, the polarization directions of the nonequilibrium accumulations and currents are not parallel or antiparallel with the magnetizations. This gives rise to interesting physics like the spin-transfer effect in spin valves. The dynamics of the magnetization [3] is then governed by the parametric torques due to the spin-polarized current and magnetic field. This spin-transfer effect was theoretically proposed [4,5] and subsequently verified in experiment [6]. Since this novel spin torque was proposed, many interesting phenomena have been studied, such as spin-wave excitation [7,8], magnetization switching and reversal [9–14], domain-wall dynamics [15,16], and magnetic solitons [17,18]. In these studies, the dynamics of the magnetization $\mathbf{M}(\mathbf{r},t)$ is described by a modified Landau-Lifshitz equation including a term for the spin-transfer torque. In a typical ferromagnet, the magnetization is rarely uniform, i.e., the magnetization has spatial dependence, and a new form of spin torque [7,15] was proposed in conducting ferromagnetic structures. With this spin torque, nonlinear excitations on the background of the ground state are studied, such as the unique features of Néel-wall motion in a nanowire [15], the kink soliton solution and the domain-wall dynamics in a biaxial ferromagnet [16], bulk spin excitations [7,15], and magnetic soliton solutions for the isotropic [17] and uniaxial anisotropic cases [18]. Nonlinear spin waves and magnetic solitons are always topics of research in confined ferromagnetic materials [19–22] due to the interaction between the spin-polarized current and local magnetization, especially the generation and detection of magnon excitation [20] in a magnetic multilayer.

From Eq. (1) one can obtain the solution for the spin current if the magnetization $\mathbf{M}(\mathbf{r},t)$ is known. For simplicity, we consider an infinitely long ferromagnetic nanowire, where the electronic current flows along the length of the wire, defined as the x direction. The z axis is taken as the direction of the uniaxial anisotropy field and the external field. Assuming the magnetization is nonuniform only along the direction of the current, the spin current in Eq. (1) can be written as

$$\mathbf{j}(x,t)=b_J\mathbf{M}(x,t), \tag{2}$$

where $b_J=Pj_e\mu_B/(eM_s)$. The spatial variation of the spin current produces a reaction torque on the magnetization given by $\tau_b\equiv\partial\mathbf{j}/\partial x=b_J\partial\mathbf{M}/\partial x$, which enters the modified Landau-Lifshitz equation as shown below. As reported in previous work, both spin-wave and soliton solutions [7,15,18] are possible for the magnetization $\mathbf{M}(x,t)$, and the spin current $\mathbf{j}$ has a periodic or pulsed form, respectively.

In the present paper, we will investigate the properties of the spin current $\mathbf{j}$ on the background of a periodic solution corresponding to the soliton solution for the magnetization on a nonlinear spin-wave background. The paper is organized as follows. In Sec. II we transform the Landau-Lifshitz equation into an equation of nonlinear Schrödinger type in the long-wavelength approximation. By means of the Darboux transformation, the soliton solution for the spin-polarized current is constructed analytically in Sec. III. In Sec. IV we discuss the properties of the solution for the spin-polarized current in detail. Section V is our conclusion.

II. DYNAMICS OF MAGNETIZATION IN A FERROMAGNETIC NANOWIRE

The dynamics of the localized magnetization is described by the modified Landau-Lifshitz equation [7,15] including a term for the spin-transfer torque,

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_s}\mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} + \tau_b, \tag{3}$$

where the localized magnetization $\mathbf{M} \equiv \mathbf{M}(x,t)$, γ is the gyromagnetic ratio, α is the Gilbert damping parameter, and $\mathbf{H}_{\text{eff}}$ represents the effective magnetic field including the external field, the anisotropy field, the demagnetization field, and the exchange field. This effective field can be written as $\mathbf{H}_{\text{eff}}=(2A/M_s^2)\partial^2\mathbf{M}/\partial x^2+[(H_K/M_s-4\pi)M_z+H_{\text{ext}}]\mathbf{e}_z$, where A is the exchange constant, H_K is the anisotropy field, H_{ext} is the applied external field, and $\mathbf{e}_z$ is the unit vector along the z direction. Introducing the normalized magnetization $\mathbf{m}=\mathbf{M}/M_s$, Eq. (3) can be simplified in the dimensionless form

$$\frac{\partial \mathbf{m}}{\partial t} = -\left(\mathbf{m} \times \frac{\partial^2 \mathbf{m}}{\partial x^2}\right) + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} + \frac{b_J t_0}{l_0}\frac{\partial \mathbf{m}}{\partial x} - \left(m_z + \frac{H_{\text{ext}}}{H_K - 4\pi M_s}\right)(\mathbf{m} \times \mathbf{e}_z), \tag{4}$$

where the time t and space coordinate x have been rescaled by the characteristic time $t_0=1/[\gamma(H_K-4\pi M_s)]$ and length $l_0=\sqrt{2A/[(H_K-4\pi M_s)M_s]}$, respectively.

It is obvious that $\mathbf{m} \equiv (m_x,m_y,m_z)=(0,0,1)$ forms the ground state of the system, and two types of nonlinear excited state, i.e., the spin-wave solution and magnetic soliton, are possible for Eq. (4). When the magnetic field is high enough, the deviation of the magnetization from the ground state is small for the two types of excited state. In this case we can make a reasonable transformation,

$$\psi = m_x + i m_y, \quad m_z = \sqrt{1-|\psi|^2}. \tag{5}$$

Substituting the above equations into Eq. (4) we obtain

$$i\frac{\partial \psi}{\partial t} = m_z\frac{\partial^2 \psi}{\partial x^2} - \psi\frac{\partial^2 m_z}{\partial x^2} - \alpha\left(m_z\frac{\partial \psi}{\partial t} - \psi\frac{\partial m_z}{\partial t}\right) + i\frac{b_J t_0}{l_0}\frac{\partial \psi}{\partial x} - \left(m_z + \frac{H_{\text{ext}}}{H_K - 4\pi M_s}\right). \tag{6}$$

It is easy to get two solutions of Eq. (6): one is $\psi=0$, corresponding to the ground state $\mathbf{m}=(0,0,1)$, i.e., $\mathbf{j}=(0,0,b_J M_s)$, and the other is the bulk spin-wave excitation $\psi=A_c e^{i(-k_c x+\omega_c t)}$, corresponding to the periodic spin current

$$j_x = b_J M_s A_c \cos(-k_c x + \omega_c t),$$

$$j_y = b_J M_s A_c \sin(-k_c x + \omega_c t),$$

$$j_z = b_J M_s \sqrt{1-A_c^2}, \tag{7}$$

where k_c and ω_c are the dimensionless wave number and frequency of the spin wave, and the transverse amplitude $A_c \ll 1$. For an attractive interaction nonlinear spin waves in a ferromagnet with anisotropy lead to macroscopic phenomena, i.e., the appearance of a spatially localized magnetic excited state (magnetic soliton).

In the present paper, we want to obtain the soliton solution of the magnetization on a nonlinear spin-wave background in a uniaxial ferromagnetic nanowire with spin torque. However, Eq. (6) is not integrable. For our purpose, we consider the case without damping and the long-wavelength approximation [23], where the dimensionless wave number $k_c \ll 1$. Keeping only the nonlinear terms of the order of magnitude of $|\psi|^2\psi$, Eq. (6) can be simplified as the integrable equation

$$i\frac{\partial \psi}{\partial t} = \frac{\partial^2 \psi}{\partial x^2} + \frac{1}{2}|\psi|^2\psi - \left(1 + \frac{H_{\text{ext}}}{H_K - 4\pi M_s}\right)\psi + i\frac{b_J t_0}{l_0}\frac{\partial \psi}{\partial x}, \tag{8}$$

whose soliton solution on the background of the ground state $\psi=0$ can be obtained by the Hirota method [18,24]. In order to discuss the properties of the soliton solution on the spin-wave background, here we use a straightforward Darboux transformation [25–27] to construct the general expression for the soliton solution of Eq. (8), with which the soliton solution for the spin-polarized current is obtained from Eqs. (2) and (5). For this reason, we will consider mainly the solution of Eq. (8) in the following section.

The main idea of the Darboux transformation is that it first transforms the nonlinear equation into the Lax representation, and then by a series of transformations the soliton solution can be constructed algebraically with the obvious seed solution of the nonlinear equation. In terms of the Ablowitz-Kaup-Newell-Segur technique, the Lax representation for Eq. (8) can be constructed as

$$\frac{\partial \Psi}{\partial x} = U\Psi,$$

$$\frac{\partial \Psi}{\partial t} = V\Psi, \tag{9}$$

where $\Psi=(\Psi_1 \;\; \Psi_2)^T$, the superscript T denotes the matrix transpose, and the Lax pairs U and V are defined by

$$U = \lambda\sigma_3 + q,$$

$$V = (-i2\lambda^2 + \lambda\alpha_1 + \alpha_2)\sigma_3 + (\alpha_1 - i2\lambda)q + i\left(\frac{\partial q}{\partial x} + q^2\right)\sigma_3, \tag{10}$$

where

$$\alpha_1 = \frac{b_J t_0}{l_0}, \quad \alpha_2 = i\frac{1}{2}\left(1 + \frac{H_{\text{ext}}}{H_K - 4\pi M_s}\right),$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad q = \frac{1}{2}\begin{pmatrix} 0 & \psi \\ -\bar{\psi} & 0 \end{pmatrix}.$$

Here λ is the complex spectral parameter, and the overbar denotes the complex conjugate. With the natural condition of Eq. (9), $\partial^2\Psi/(\partial x\,\partial t)=\partial^2\Psi/(\partial t\,\partial x)$, i.e., $\partial U/\partial t-\partial V/\partial x+[U,V]=0$, the integrable Eq. (8) can be recovered successfully. Now, Eqs. (9) and (10) comprise the normal form of the developed Darboux transformation with which we can get the general N-soliton solution as shown in the next section.

III. DARBOUX TRANSFORMATION

In this section, we briefly introduce the procedure for getting the soliton solution for the developed Darboux transformation. Consider the following transformation:

$$\Psi[1]=(\lambda I-K)\Psi, \tag{11}$$

where $K=S\Lambda S^{-1}$, $\Lambda=\mathrm{diag}(\lambda_1,\lambda_2)$, and S is a nonsingular matrix that satisfies

$$\frac{\partial}{\partial x}S=\sigma_3 S\Lambda+qS. \tag{12}$$

We let $\Psi[1]$ satisfy the Lax equation

$$\frac{\partial}{\partial x}\Psi[1]=U_1\Psi[1], \tag{13}$$

where

$$U_1=\lambda\sigma_3+q_1,$$

$$q_1=\frac{1}{2}\begin{pmatrix}0 & \psi_1\\ -\bar{\psi}_1 & 0\end{pmatrix}.$$

With the help of Eqs. (10)–(12), we obtain the Darboux transformation from Eq. (13) in the form

$$\psi_1=\psi+4K_{12}, \tag{14}$$

which shows that a new solution ψ_1 of Eq. (8) with the seed solution ψ can be obtained if K is known.

It is easy to verify that, if $\Psi=(\Psi_1\ \ \Psi_2)^T$ is an eigenfunction of Eq. (9) with eigenvalue $\lambda=\lambda_1$, then $(-\bar{\Psi}_2\ \ \bar{\Psi}_1)^T$ is also an eigenfunction, but with eigenvalue $-\bar{\lambda}_1$. Therefore, S and Λ can be taken in the form

$$S=\begin{pmatrix}\Psi_1 & -\bar{\Psi}_2\\ \Psi_2 & \bar{\Psi}_1\end{pmatrix},\quad \Lambda=\begin{pmatrix}\lambda_1 & 0\\ 0 & -\bar{\lambda}_1\end{pmatrix}, \tag{15}$$

which ensures that Eq. (12) holds. Then Eq. (14) becomes

$$\psi_1=\psi+4(\lambda_1+\bar{\lambda}_1)\frac{\Psi_1\bar{\Psi}_2}{\Psi^T\bar{\Psi}}, \tag{16}$$

where $\Psi^T\bar{\Psi}=|\Psi_1|^2+|\Psi_2|^2$, $\Psi=(\Psi_1,\Psi_2)^T$. We wish to determine the eigenfunction of Eq. (9) corresponding to the eigenvalue λ_1 for the solution ψ of Eq. (8). Thus, by solving Eq. (9),we can generate a new solution for $\mathbf{j}$ with the help of Eqs. (2), (5), and (8) from an obvious seed solution of Eq. (8).

To obtain the exact N-order solution, we first rewrite the Darboux transformation in Eq. (16) as

$$\psi_1=\psi+4(\lambda_1+\bar{\lambda}_1)\frac{\Psi_1[1,\lambda_1]\bar{\Psi}_2[1,\lambda_1]}{\Psi[1,\lambda_1]^T\bar{\Psi}[1,\lambda_1]}, \tag{17}$$

where $\Psi[1,\lambda]=(\Psi_1[1,\lambda],\Psi_2[1,\lambda])^T$ denotes the eigenfunction of Eq. (9) corresponding to the eigenvalue λ. Then, repeating the above procedure N times, we can obtain the exact N-order solution

$$\psi_N=\psi+4\sum_{n=1}^{N}(\lambda_n+\bar{\lambda}_n)\frac{\Psi_1[n,\lambda_n]\bar{\Psi}_2[n,\lambda_n]}{\Psi[n,\lambda_n]^T\bar{\Psi}[n,\lambda_n]}, \tag{18}$$

where

$$\Psi[n,\lambda]=(\lambda-K[n-1])\cdots(\lambda-K[1])\Psi[1,\lambda],$$

$$K_{l_1l_2}[n']=(\lambda_{n'}+\bar{\lambda}_{n'})\frac{\Psi_{l_1}[n',\lambda_{n'}]\overline{\Psi}_{l_2}[n',\lambda_{n'}]}{\Psi[n',\lambda_{n'}]^T\overline{\Psi}[n',\lambda_{n'}]}-\bar{\lambda}_{n'}\delta_{l_1l_2}.$$

Here $\Psi[n',\lambda]$ is the eigenfunction corresponding to $\lambda_{n'}$ for $\Psi_{n'-1}$ with $\Psi_0\equiv\Psi$, and $l_1,l_2=1,2$, $n'=1,2,\ldots,n-1$, $n=2,3,\ldots,N$. Thus, after choosing a seed as the basic initial solution, by solving the linear characteristic equation system (9), one can construct a set of new solutions from Eq. (18).

In the following, we take the initial seed solution $\psi=A_c e^{i(-k_cx+\omega_ct)}$ corresponding to a periodic solution (7), where the dispersion relation $\omega_c=k_c^2-k_cb_Jt_0/l_0-A_c^2/2+[1+H_{\mathrm{ext}}/(H_K-4\pi M_s)]$ is obtained from Eq. (8). After a tedious calculation for solving the linear equation system (9), we have the eigenfunction corresponding to the eigenvalue λ in the form

$$\Psi_1=LC_1e^{\Theta_1}+\frac{1}{2}A_cC_2e^{\Theta_2},$$

$$\Psi_2=\frac{1}{2}A_cC_1e^{-\Theta_2}+LC_2e^{-\Theta_1}, \tag{19}$$

where the parameters C_1 and C_2 are arbitrary complex constants, and the other parameters are defined by

$$\Theta_1=-\frac{1}{2}i(k_cx-\omega_ct)+D(x+\delta t),$$

$$\Theta_2=-\frac{1}{2}i(k_cx-\omega_ct)-D(x+\delta t),$$

$$L=-\frac{1}{2}ik_c-D-\lambda,$$

$$D=\frac{1}{2}\sqrt{(ik_c+2\lambda)^2-A_c^2},$$

$$\delta=-i2\lambda-k_c+\frac{b_Jt_0}{l_0}. \tag{20}$$

With the help of the formulas (2), (5), (18), and (19) we can obtain the desired soliton solution for the spin current.

IV. PROPERTIES OF THE SOLITON SOLUTION FOR THE SPIN-POLARIZED CURRENT

Taking the spectral parameter $\lambda=\lambda_1\equiv\mu_1/2+i\nu_1/2$, where μ_1 and ν_1 are real numbers, in Eq. (19), and substituting into Eqs. (17) and (5), we obtain the one-soliton solution for the spin-polarized current from Eq. (2) as

$$j_x = b_J M_s\left(A_c \cos\varphi + \frac{2\mu_1}{\Delta_1}(Q_1\cos\varphi - Q_2\sin\varphi)\right),$$

$$j_y = b_J M_s\left(A_c \sin\varphi + \frac{2\mu_1}{\Delta_1}(Q_1\sin\varphi + Q_2\cos\varphi)\right),$$

$$j_z = b_J M_s\sqrt{1-\left(A_c+\frac{2\mu_1 Q_1}{\Delta_1}\right)^2-\left(\frac{2\mu_1 Q_2}{\Delta_1}\right)^2}, \qquad (21)$$

where

$$\theta_1 = 2D_{1R}x + 2(D_1\delta_1)_R t + 2x_0,$$

$$\Phi_1 = 2D_{1I}x + 2(D_1\delta_1)_I t - 2\varphi_0,$$

$$\varphi = -k_c x + \omega_c t, \qquad (22)$$

$$Q_1 = A_c L_{1R}\cosh\theta_1 + \left(|L_1|^2+\frac{1}{4}A_c^2\right)\cos\Phi_1,$$

$$Q_2 = A_c L_{1I}\sinh\theta_1 + \left(|L_1|^2-\frac{1}{4}A_c^2\right)\sin\Phi_1,$$

$$\Delta_1 = \left(|L_1|^2+\frac{1}{4}A_c^2\right)\cosh\theta_1 + A_c L_{1R}\cos\Phi_1,$$

where the subscript R and I represent the real and imaginary parts, respectively. The other parameters are $D_1=\sqrt{(ik_c/2+\lambda_1)^2-A_c^2/4}$, $L_1=-ik_c/2-D_1-\lambda_1$, $\delta_1=-i2\lambda_1-k_c+b_J t_0/l_0$, $x_0=-(\ln|C_2/C_1|)/2$, and $\varphi_0=[\arg(C_2/C_1)]/2$, where C_1, C_2 are arbitrary complex constants.

The solution (21) describes a one-soliton solution for the spin-polarized current in a ferromagnetic nanowire embedded in the periodic spin current background (7). (a) When $\mu_1=0$, the solution (21) reduces to the periodic solution (7). (b) When the spin-wave amplitude vanishes, namely, $A_c=0$, the solution (21) reduces to a solution in the form

$$j_x = \frac{2\mu_1 b_J M_s}{\cosh\theta_1}\cos(\Phi_1+\eta),$$

$$j_y = \frac{2\mu_1 b_J M_s}{\cosh\theta_1}\sin(\Phi_1+\eta),$$

$$j_z = b_J M_s\sqrt{1-\frac{4\mu_1^2}{\cosh^2\theta_1}}, \qquad (23)$$

where

$$\theta_1 = \mu_1\left[x+\left(2\nu_1+\frac{b_J t_0}{l_0}\right)t+\frac{2}{\mu_1}x_0\right],$$

$$\Phi_1 = \nu_1\left\{x-\left[\frac{1}{\nu_1}(\mu_1^2-\nu_1^2)-\frac{b_J t_0}{l_0}\right]t-\frac{2}{\nu_1}\varphi_0\right\},$$

$$\eta = \left(1+\frac{H_{\rm ext}}{H_K-4\pi M_s}\right)t. \qquad (24)$$

The solution (23) is in fact the same as the solution (8) in Ref. [18]. [One should notice that the transformation (5) is different from that in Ref. [18].]

The solution (23) indicates a spatially localized excitation [28], which is denoted by the transverse amplitude $2\mu_1$ of the deviation from the ground state $\mathbf{j}=(0,0,b_J M_s)$. The components j_x and j_y precess around the component j_z with the frequency $\Omega_1=\nu_1^2-\mu_1^2+b_J t_0/l_0+1+H_{\rm ext}/(H_K-4\pi M_s)$, and the soliton solution is characterized by the width $1/\mu_1$ and the velocity of the soliton center $v_1=-(2\nu_1+b_J t_0/l_0)$. The wave number $k_{s,1}=-\nu_1$ and the frequency Ω_1 of the "carrier wave" are related by the dispersion law $\Omega_1=k_{s,1}^2-k_{s,1}b_J t_0/l_0-\mu_1^2+1+H_{\rm ext}/(H_K-4\pi M_s)$, which shows that the magnetic field contributes to the precession frequency only. The magnetic soliton energy is seen to be $E_1=-b_J^2 t_0^2/(2l_0)^2-\mu_1^2+1+H_{\rm ext}/(H_K-4\pi M_s)+\frac{1}{2}m^* v_1^2$, where the dimensionless effective mass m^* of the soliton is 1/2.

From Eqs. (23) and (24), we also see that the term b_J can change the velocity and the precessional frequency of the soliton on the background of the ground state $\mathbf{j}=(0,0,b_J M_s)$. This case confirms the previous studies [15,18]. However, on the background (7) unusual properties of Eq. (21) will be described below. The properties of the envelope soliton solution (21) are characterized by the width $1/(2D_{1R})$, the wave number $k_s=-2D_{1I}$, the initial center position $-x_0/D_{1R}$, and the envelope velocity $v_1=-(D_1\delta_1)_R/D_{1R}$. The initial center position of the soliton is moved $x_0(2/\mu_1-1/D_{1R})$ by the spin wave, which shows a different way of controlling the soliton in space.

With the expressions of D_1 and δ_1, we find that the velocity and the width of the envelope soliton are modulated by the amplitude A_c and wave number k_c of the spin wave as shown in Fig. 1. From Figs. 1(a) and 1(c), we see that the absolute value of the velocity and the width of the envelope soliton become larger with increasing A_c. Figure 1(b) shows that the value of k_c near $-\nu_1$ has an obvious effect on the velocity of the soliton. When $k_c=-\nu_1$, the width of the soliton is maximal, as shown in Fig. 1(d).

From Eq. (22) we can directly see that, when $D_{1I}\delta_{1I}=\delta_{1R}D_{1R}$, the parameter θ_1 depends only on x, which implies that the envelope velocity $-(D_1\delta_1)_R/D_{1R}$ becomes zero, i.e., the soliton is trapped in space by the nonlinear spin wave. It should be noted that this condition can be written as

$$j_e = \frac{eM_s}{P\mu_B}\frac{l_0}{t_0}\left(-\mu_1\frac{D_{1I}}{D_{1R}}-\nu_1+k_c\right), \qquad (25)$$

which is determined by the characteristic velocity l_0/t_0, the amplitude and the wave number of the soliton and the nonlinear spin wave, and the parameter $(eM_s/P\mu_B)$. After a tedious calculation, we found that when $A_c\ll k_c$ and $\mu_1\ll\nu_1$ the condition in Eq. (25) reduces to $j_e\approx-2\nu_1(l_0/t_0)(eM_s/P\mu_B)$. When the amplitude of the spin wave van-

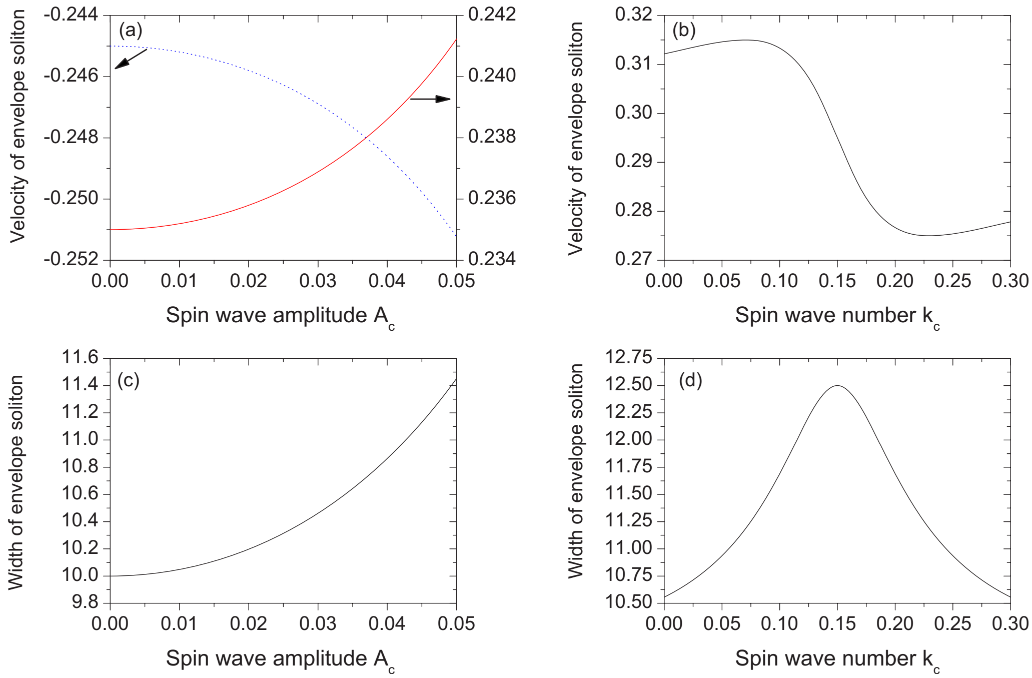

FIG. 1. (Color online) Velocity and width of the soliton solution for spin-polarized current vs the amplitude A_c and wave number k_c of the spin wave. The parameters are $\mu_1=0.1$, $l_0=2\times10^{-8}$ cm, $t_0=5.7392\times10^{-12}$ s, and $b_J=52$ cm/s. (a) Velocity vs amplitude A_c: $k_c=0.1$, $\nu_1=-0.12$ (red solid line); $k_c=-0.1$, $\nu_1=0.12$ (blue dotted line). (b) Velocity vs spin-wave number k_c; $A_c=0.06$, $\nu_1=-0.15$. (c) Width vs amplitude A_c; $k_c=-0.1$, $\nu_1=0.12$. (d) Width vs spin-wave number k_c; $A_c=0.06$, $\nu_1=-0.15$.

ishes, namely, $A_c=0$, the trapping condition in Eq. (25) reduces to $j_e=-2\nu_1(l_0/t_0)(eM_s/P\mu_B)$, which is determined by the characteristic velocity l_0/t_0, the soliton wave number ν_1, and the parameter $(eM_s/P\mu_B)$. These results show that the background has almost no effect on the trapping conditions in the special case $A_c \ll k_c$ and $\mu_1 \ll \nu_1$. For Co_3Pt alloy films [21], which have high perpendicular anisotropy, we chose $H_K=1\times10^4$ Oe, $A=1.0\times10^{-6}$ erg/cm, $4\pi M_s=1\times10^2$ Oe, $\gamma=1.76\times10^7$ Oe^{-1} s^{-1}, $P=0.35$, and the dimensionless parameters $k_c=0.05$, $A_c=0.02$, $\nu_1=-0.12$, and $\mu_1=0.1$. The critical electric current trapping the soliton is $j_e=1.867\times10^4$ A/cm^2. It is very important to point out that, from Eqs. (21) and (22) and the expression for δ_1, the term b_J changes not only the velocity but also the frequency, affecting the soliton energy on the background (7). This property trapping the soliton in space by the nonlinear spin wave is characterized by spatial and temporal periods along the direction of soliton propagation $x=-(D_1\delta_1)_R t/D_{1R}-x_0/D_{1R}$ given by $\pi(D_1\delta_1)_R/[\delta_{1I}(D_{1R}^2+D_{1I}^2)]$ and $\pi D_{1R}/[\delta_{1I}(D_{1R}^2+D_{1I}^2)]$, respectively.

In order to explain some interesting properties of the solution (21),we discuss the special case $k_c=-\nu_1$ and analyze two representative situations in detail. (a) The amplitude A_c exceeds one-half of the transverse amplitude $2\mu_1$ of the soliton. (b) The amplitude A_c is less than one-half of the transverse amplitude $2\mu_1$ of the soliton (which implies $A_c,\mu_1>0$).

(a) In the case $\mu_1^2<A_c^2$, the solution (21) reduces to

$$j_x = b_J M_s (R_1' \cos\varphi - R_2' \sin\varphi),$$

$$j_y = b_J M_s (R_1' \sin\varphi + R_2' \cos\varphi),$$

$$j_z = b_J M_s \sqrt{1 - A_c^2 - \frac{\zeta_1 (A_c \cosh\theta_1 \cos\Phi_1 - \mu_1)}{(A_c \cosh\theta_1 - \mu_1 \cos\Phi_1)^2}}, \quad (26)$$

where φ is given in Eq. (22), and the other parameters are determined by

$$\zeta_1 = 4\mu_1\kappa_1^2,$$

$$\kappa_1 = \sqrt{A_c^2 - \mu_1^2},$$

$$R_1' = -A_c + \frac{2\kappa_1^2 \cosh\theta_1}{A_c\cosh\theta_1 - \mu_1\cos\Phi_1},$$

$$R_2' = -\frac{2\mu_1\kappa_1 \sinh\theta_1}{A_c\cosh\theta_1 - \mu_1\cos\Phi_1},$$

$$\theta_1 = \mu_1\kappa_1 t + 2x_0,$$

$$\Phi_1 = \kappa_1(x - v_1 t - 2\varphi_0/\kappa_1). \quad (27)$$

A simple analysis of Eq. (27) reveals that the solution (26) is periodic in the space coordinate, denoted by $2\pi/\kappa_1$, and aperiodic in the temporal variable, as shown in Fig. 2. From Fig. 2 we can see that the background becomes unstable; therefore the solution (26) can be considered as describing the modulation instability process [29]. Along the propagation direction of the soliton, the expression for j_z has a maximum $j_z=b_J M_s$, i.e., $m_z=1$, at $\cos\Phi_1=(2\mu_1^2-A_c^2)/\mu_1 A_c$ when

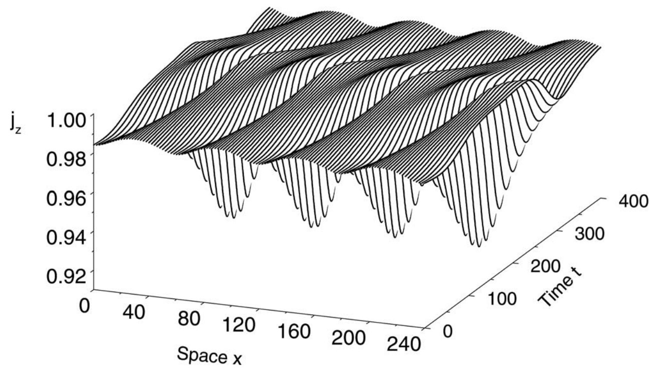

FIG. 2. Evolution of solution (26) with conditions $k_c=-\nu_1$, $\mu_1^2 < A_c^2$. Parameters are $\mu_1=0.12$, $\nu_1=-0.1$, $A_c=0.16$, $b_J=34.8$ cm/s, $l_0=2\times 10^{-10}$ m, $t_0=5.7392\times 10^{-12}$ s, $x_0=-1.27$, and $\varphi_0=0$. The spin current j_z is in units of $b_J M_s$, here and in Figs. 3 and 4.

$A_c^2/4 < \mu_1^2 < A_c^2$, which shows that these points are not excited even on the spin-wave background, and it has a minimum $j_z=b_J M_s[1-(2\mu_1+A_c)^2]^{1/2}$ at $\sin\Phi_1=0$. When $\mu_1^2 < A_c^2/4$, the expression for j_z has a maximum $j_z=b_J M_s[1-(2\mu_1-A_c)^2]^{1/2}$ at $\Phi_1=0$, and a minimum $j_z=b_J M_s[1-(2\mu_1+A_c)^2]^{1/2}$ at $\Phi_1=\pi$. These results show that the linear combined transverse amplitude of the spin wave and magnetic soliton can be obtained in these special cases.

(b) In the case $\mu_1^2 > A_c^2$, the solution (21) reduces to

$$j_x = b_J M_s (R_1 \cos\varphi - R_2 \sin\varphi),$$

$$j_y = b_J M_s (R_1 \sin\varphi + R_2 \cos\varphi),$$

$$j_z = b_J M_s \sqrt{1 - A_c^2 - \frac{\zeta_2(\mu_1 - A_c \cosh\theta_1 \cos\Phi_1)}{(\mu_1 \cosh\theta_1 - A_c \cos\Phi_1)^2}}, \quad (28)$$

where

$$\zeta_2 = 4\mu_1\kappa_2^2,$$

$$\kappa_2 = \sqrt{\mu_1^2 - A_c^2}, \quad (29)$$

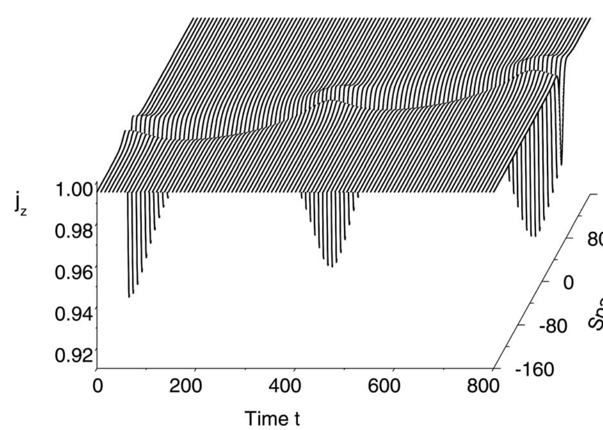

FIG. 3. Evolution of solution (28) with the conditions $k_c=-\nu_1$ and $\mu_1^2 > A_c^2$. Parameters are $\mu_1=0.1$, $\nu_1=-0.08$, $A_c=0.06$, $b_J=41.8$ cm/s, $l_0=2\times 10^{-10}$ m, $t_0=5.7392\times 10^{-12}$ s, $x_0=3$, and $\varphi_0=0$.

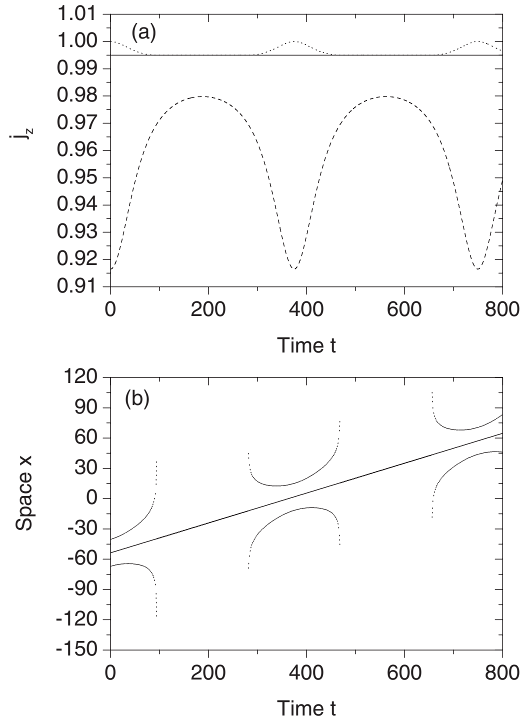

FIG. 4. (a) Evolution of the minimum amplitude of j_z in Eq. (31) (dashed line), the maximum amplitude of j_z in Eq. (32) (dotted line), and the background amplitude of j_z (solid line) in Eq. (7). (b) Location of the minimum amplitude (solid line) and the maximum amplitude (dotted line) in the time–propagation-distance plane. The parameters are the same as in Fig. 3.

$$R_1 = -A_c + \frac{2\kappa_2^2 \cos\Phi_1}{\mu_1 \cosh\theta_1 - A_c \cos\Phi_1},$$

$$R_2 = \frac{2\mu_1\kappa_2 \sin\Phi_1}{\mu_1 \cosh\theta_1 - A_c \cos\Phi_1},$$

$$\theta_1 = \kappa_2(x - v_1 t + 2x_0/\kappa_2),$$

$$\Phi_1 = -\mu_1\kappa_2 t - 2\varphi_0. \quad (30)$$

With the expressions (28) and (30), we can see the main characteristics of the soliton solution. (1) The soliton has the same envelope velocity $v_1=-(2\nu_1+b_J t_0/l_0)$ on both the background of the periodic spin current in Eq. (7) and the ground state background $\mathbf{j}=(0,0,b_J M_s)$. (2) The amplitude of j_z in Eq. (28) has a temporal periodic oscillation as shown in Fig. 3. A detailed calculation shows that the amplitude of j_z in Eq. (28) has a minimum at $\theta_1=0$, which is given by

$$j_z = b_J M_s \sqrt{1 - A_c^2 - \frac{4\mu_1(\mu_1^2 - A_c^2)}{\mu_1 - A_c \cos\Phi_1}}, \quad (31)$$

and has a maximum

$$j_z = b_J M_s \sqrt{1 - \frac{\mu_1^2 A_c^2 \sin^2\Phi_1}{\mu_1^2 - A_c^2 \cos^2\Phi_1}}, \quad (32)$$

at $\cosh\theta_1 = 2\mu_1/(A_c\cos\Phi_1) - (A_c\cos\Phi_1)/\mu_1$. Figure 4(a) presents the evolution along the propagation direction of the minimum and maximum intensities given by Eqs. (31) and (32) (see, respectively, the dashed and dotted lines), and the spin-wave intensity (solid line). The location of the minimum and maximum amplitudes (solid and dotted lines, respectively) in the time-space plane is shown in Fig. 4(b). From Fig. 4, it is seen that the narrower the soliton, the sharper the peak and the deeper the two dips at the wings of the soliton. This feature illustrates the characteristic breather behavior of the soliton as it propagates on the background of a periodic solution for the spin current in a ferromagnetic nanowire.

V. CONCLUSION

In summary, by transforming the modified Landau-Lifshitz equation into an equation of nonlinear Schrödinger type, we study the interaction of a periodic and a one-soliton solution for the spin-polarized current in a uniaxial ferromagnetic nanowire. Our results show that the amplitude of the soliton solution has a spatial and temporal period on the background of a periodic spin current. The effective mass of the soliton is obtained. Moreover, we found that the soliton can be trapped only in space. We also analyze the modulation instability and dark soliton on the background of a periodic spin current. This soliton solution shows the characteristic breather behavior of the soliton as it propagates along the ferromagnetic nanowire.

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